The Phase in a Quantum-Dot Interferometer Modulated by Rashba Spin-Orbit Interaction

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***Abstract*—Considering Rashba spin-orbit interaction (SOI) in a interferometer with a quantum dot (QD), the phase for spin-polarized electrons transport through QD is systemically studied. The results show that, due to the presence of SOI, the spin-polarized electrons no longer obey the Onsager symmetry: $G_\sigma(\varphi) \neq G_\sigma(-\varphi)$ neither of the up-spin and down-spin conductances are the even functions as magnetic flux, i.e. . Moreover, the Aharonov-Bohm AB phase difference between up-spin and down-spin electrons is proportional to the spin precession angle. In contrast to this, the transmission phase for up-spin and down-spin electrons traversing the quantum dot presents similar behaviors, which continuously increase by π as the energy of the dot level is tuned through a resonance, and presents a phase lapse accompanying transmission zero. These novel phenomena may be originated from the intrinsic effect of electric field on electron transport.**

***Index Terms*—Aharonov-Bohm Phase; transmission phase; quantum dot; Rashba spin-orbit interaction**

I. Introduction

Since Yacoby et al. [1] initially demonstrated that adopting the structure in which one quantum dot (QD) embedded in the arm of Aharonov-Bohm (AB) ring, the properties of electron coherent transport through QD can be observed, there has been considerable interest in the problem of coherent transport through an interacting mesoscopic system both in experiment and theory. Experimentally, it was firstly found [1] that in a two-terminal and closed configuration (a) all the successive resonances are in phase; (b) the phase changes by π abruptly across the resonance, namely, "phase rigidity". Next, in a double-slit-like interference experiment with a four-terminal configuration, R. Schuster et al. [2] observed that: (a) the phase changes by π continuously across the resonance; (b) between the adjacent resonance the phase suddenly lapses π ; (c) all the resonance are in phase.

The sudden phase drop and in-phase resonances were later reproduced in other experiments [3]-[5]. Very recently, Kobayashi et al. [6] observed that although the conductance of a closed Aharonov-Bohm interferometer with a quantum dot on one branch, obeys the Onsager symmetry under magnetic field reversal, it needs not be a periodic function of this field: The conductance maxima move with both the field and the gate voltage on the dot, presenting an apparent breakdown of "phase rigidity." The results indicate that it is possible to obtain the real phase change for electron transport through QD in the two-terminal and closed structures. Theoretically, based on the Friedel sum rule [7], it can be well understood for the in-phase behavior of all the resonances [8]. Lately, a mass of investigations focus on the phase lapse characteristic [9]-[12]. However, so far, about above stated phenomena, there is still no fully satisfactory framework has been established yet. In particular, with regard to the coherent transport of the spin-polarized electrons, few studies are done except for the Kondo correlation [13]-[15]. Generally, for the quantum information processing and computing devices, the spin degree of freedom of an electron cannot be neglected. Meanwhile, the spin-orbit interaction (SOI) in mesoscopic system has also attracted great attentions in recent years [16]–[23], as it plays a very interesting role in the growing field of spintronics that intends to explore the electron spin to store and transport information [24]-[25]. For SOI, the magnitude of it can be controlled and manipulated by external electric field [17]-[18], and it also can couple the spin degree of freedom of an electron to its orbital motion and vice versa. Thereby these giving a useful handle for manipulating and controlling the electron spin by external electric fields or gate voltages. Moreover, if SOI is taken into consideration, the usage of any magnetic materials or fields to obtain spin polarization will be avoided. However, the previous works only focus on how to improve the spin polarization efficiency based on SOI. For example, Murayama et al. [19] experimentally studied the spin polarization efficiency in a vertical double QD device, and reported that it could reach 40%. So far, there still have few studies on the phase of spin-polarizd electron with the help of SOI in mesoscopic system. Actually, due to the SOI, in the Rashba effect which arises from the inversion asymmetry of the electric field in a two-dimensional electron system, up-spin and down-spin electrons traveling through the upper and lower arms will acquire different phases, which result in an interesting spin-relevant phase coherence phenomenon appearing.

In this paper, using the structure in which one quantum dot (QD) embedded in the arm of AB ring, we research the SQI and find out that the presence of SOI not only leads the up-spin and down-spin electrons separated, that is, spin polarized, but also controls the transmission phase for spin-polarized electrons transport through QD. Utilizing this idea, it may be feasible to detect up-spin and down-spin electrons by manipulating the strength of SOI on experiments.

Manuscript received August 27, 2011. This work is supported by the Beijing Excellent Talent, China (Grant No. 2009D005016000019), the Foundation of Beijing Educational Commission, China (Grant No. KM201010028006) and the National Natural Science Foundation of China (Grant No. 10874124).

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II. THEORETICAL MODELS AND FORMULAE

Our model is shown in Fig. 1; an AB ring with a QD embedded in the lower arm is coupled to two leads. The AB flux Φ through the AB frame is introduced by a homogeneous magnetic field B. The electric field is orthogonal to the ring, which tunes the Rashba SOI strength α_R in the experiment. The essence of the SOI is the interaction between the external electric field and the moving spins. It gives to a spin precession [21]. The spin precession angle can be described as $\theta = \alpha_R m^* L/\hbar^2$, with m^* being the electron effective mass and L being the size of QD. In the present device, due to the Rashba SOI, an extra phase is generated when an electron transports from QD to right lead [26]-[27]. In this case, the Hamiltonian of this system is expressed as:

$$
\begin{aligned}
H &= \sum_{\beta=l,r} H_\beta + H_d + H_t \\
&= \sum_{\beta=l,r} \varepsilon_{\beta k} c^+_{\beta k\sigma} c_{\beta k\sigma} + \sum_{\sigma} \varepsilon_d d^+_\sigma d_\sigma + U n_{\bar{\sigma}} n_\sigma \\
&+ \left[T_{lr} \sum_{\sigma} c^+_{lk\sigma} c_{rk\sigma} + T_{ld} \sum_{\sigma} c^+_{lk\sigma} d_\sigma + T_{rd\sigma} \sum_{\sigma} c^+_{rk\sigma} d_\sigma . \right] + H.c
\end{aligned}
\tag{1}
$$

where the first term $H_\beta = \varepsilon_{\beta k} c^+_{\beta k\sigma} c_{\beta k\sigma}$ $(\beta = l, r)$ describes the Hamiltonian of the left and right lead, respectively. $\varepsilon_{\beta k}$ is the single-electron energy, and $c^+_{\beta k\sigma}$ ($c_{\beta k\sigma}$) is the creation (annihilation) operator of the electrons in the leads. The second term H_d corresponds to the isolated QD, where ε_d is the single-particle energy level in the QD, σ denotes the state of spin, $\sigma = 1$ for up spin, and $\sigma = -1$ for down spin, $d^+_\sigma (d_\sigma)$ is creation (annihilation) operator in the QD, U is the on-site Coulomb repulsion, $n_{\sigma(\bar{\sigma})}$ is the intradot occupation number of state $\sigma(\bar{\sigma})$ in the QD. The last term in Eq.1 represents the tunneling of electrons transporting through the arm of AB ring and the QD. $T_{ld} = t_{ld}$, $T_{lr} = t_{lr}$, $T_{rd\sigma} = t_{rd} \exp(-i\sigma\theta + i\varphi)$, where t_{ij} denotes the hopping matrix element between individual parts of i and j in the absence of the magnetic flux, $\varphi = (2\pi/\Phi_0)\int A \cdot dl = 2\pi\Phi/\Phi_0$ represents the effect of magnetic flux Φ enclosed by AB ring on the phase of electron wave function, where $\Phi_0 = h/e$ is the magnetic quantum (h is Planck's constant, e is the charge of electron), A is the vector potential. In the term $T_{rd\sigma} = t_{rd} \exp(-i\sigma\theta + i\varphi)$, there is a spin-dependent phase factor $-\sigma\theta$ as a result of Rashba SOI. Here the external electric field is uniform along the ring loop and the Aharonov-Casher phases will be not considered, therefore, the spin-dependent phase enter in Eq.(1) only considering a simple facto $\sigma\theta$.

In this work, the theoretical analysis to the system is based on the standard Keldysh non-equilibrium Green functions (NEGF) theory, which is an effective way to solve many-body correlations and interactions in a unified fashion.

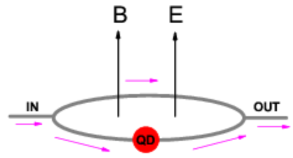

Fig. 1. Schematic diagram for the system of a QD coupled to an AB ring. The QD is embedded in the lower arm of the ring. The AB flux Φ through the AB frame is introduced by a homogeneous magnetic field B. The electric field E is orthogonal to the ring, which tunes the Rashba SOI strength α_R in experiments.

The current in the spin state of σ flowing through the AB ring takes the following form [28]:

$$
I_\sigma = \frac{2e}{\hbar} \int \frac{d\varepsilon}{2\pi} \mathrm{Re}\left[T_{ld} G^<_{dl\sigma}(\varepsilon) + T_{lr} G^<_{rl\sigma}(\varepsilon) \right] \tag{2}
$$

where $G^<_{dl\sigma}(\varepsilon)$, $G^<_{rl\sigma}(\varepsilon)$ are the Keldysh Green function, which can be obtained by the equations of motion and Dyson equations,

$$
\begin{aligned}
G^<_{dl\sigma}(\varepsilon) = \gamma \cdot \Bigg\{ & \frac{2if_l(\varepsilon)}{\pi\rho} \left(\frac{T^*_{lr} T^*_{rd\sigma}}{g^r_{dd\sigma} \cdot i\pi\rho} + \frac{T^*_{ld}}{g^r_{dd\sigma} \cdot \pi^2 \rho^2} + \frac{T_{lr} T^*_{lr} T^*_{ld}}{g^r_{dd\sigma} \cdot i\pi\rho} - T_{rd\sigma} T^{*2}_{rd\sigma} T^*_{lr} \right) \\
& - \frac{2if_r(\varepsilon)}{\pi\rho} \left(\frac{T_{lr} T^*_{rd\sigma}}{g^r_{dd\sigma} \cdot i\pi\rho} - \frac{T^2_{lr} T^*_{ld}}{g^r_{dd\sigma}} + \frac{T^2_{rd\sigma} T_{ld}}{i\pi\rho} - T_{ld} T^{*2}_{ld} T_{lr} T^*_{rd\sigma} \right) \Bigg\}
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
G^<_{rl\sigma}(\varepsilon) = \gamma \cdot \Bigg\{ & \frac{2if_l(\varepsilon)}{\pi\rho} \left(\frac{-T^*_{lr} T_{rd\sigma}}{g^r_{dd\sigma} \cdot i\pi\rho} + \frac{-T^*_{lr}}{g^{r\,2}_{dd\sigma} \cdot i\pi\rho} + \frac{T_{rd\sigma} T^*_{rd\sigma} T^*_{lr}}{g^r_{dd\sigma}} + T^2_{rd\sigma} T^*_{rd\sigma} T^*_{ld} \right) \\
& - \frac{2if_r(\varepsilon)}{\pi\rho} \left(\frac{-T_{ld} T^*_{rd\sigma}}{g^r_{dd\sigma} \cdot i\pi\rho} + \frac{-T_{lr}}{g^{r\,2}_{dd\sigma} \cdot i\pi\rho} - \frac{T_{lr} T^2_{ld}}{g^r_{dd\sigma}} - T^2_{ld} T^*_{ld} T^*_{rd\sigma} \right) \Bigg\}
\end{aligned}
\tag{4}
$$

where γ is an AB phase-dependent factor,

$$
\gamma = \left\{ \left(\frac{1}{g^r_{dd\sigma} \cdot \pi^2 \rho^2} + \frac{t^2_{lr}}{g^r_{dd\sigma}} + 2 t_{lr} t_{rd} t_{ld} \cos(\varphi) \right)^2 + \left(\frac{t^2_{ld} + t^2_{rd}}{\pi\rho} \right)^2 \right\}^{-1} \tag{5}
$$

$f_\beta(\varepsilon) = 1 / \left[e^{(\varepsilon - \mu_\beta)/k_B T} + 1 \right]$ is the Fermi distribution function in lead β, μ_β is the chemical potential, ρ is the density of state in the leads connected with AB ring, and $g^r_\sigma(\varepsilon)$ is the Green's function of the system without coupling between the leads and QD, within the wideband limit approximation [29] and the Hartree-Fock approximation[30], which can be problematic at low temperatures but should be adequate at temperatures of order of the tunneling width, which is the case of the present study (In present case, the temperature is taken to be of order Gamma which is sufficiently high relative to Kondo temperature.), the retarded Green's function $g^r_{dd\sigma}(\varepsilon)$ can be easily obtained:

$$
g^r_{dd\sigma}(\varepsilon) = \frac{1 - \langle n_{\bar{\sigma}} \rangle}{\varepsilon - \varepsilon_d} + \frac{\langle n_{\bar{\sigma}} \rangle}{\varepsilon - \varepsilon_d - U} \tag{6}
$$

where $\langle n_{\bar{\sigma}} \rangle$ is the intradot electron occupation number at state $\bar{\sigma}$, which can be calculated self-consistently [31]. The conductance of the system can be described as:

$$G_\sigma = dI_\sigma / dV \qquad (7)$$

Before carrying out the analysis of our results, we want to declare that in Ref. 22 and Ref. 26 at the last part, the works of Sun et al. focused on how to effectively polarize and manipulate the electron spin in this device based on the combined effect of a magnetic flux and the Rashba SOI. However, in our present paper, we are greatly interested in the coherent transport characteristics of spin-polarized electron in the structure, and mainly pay attention to the interference effect and the phase of spin-polarized electrons modulated by SOI, which is rarely studied by now, but it is valuable and prerequisite to concentrate in the quantum computing devices [24]-[25]. Moreover, the model Sun et al. adopted in Ref. 22 is non-adiabatic, because there is a bias between the left and right leads. but the model in our manuscript is adiabatic. In addition, the Rashba coupling should appear in the path of electrons transport. In our model electrons transport from left lead to right lead along the up and down arms of the AB ring. It means that the Rashba coupling appears in the two arms of AB ring. However, if the strength of Rashba coupling between two arms is the same, the phase factor induced by Rashba coupling will disappear. Namely, only when the phase factor induced by Rashba coupling is not zero, the difference of Rashba coupling between two arms is not zero too. So for the sake of simplicity, we assumed that the Rashba coupling in the up arm of AB ring is zero, which in the down arm contains a quantum dot is modulated by external electric field.

III. Results and Discussions

A. Fano effect and AB oscillation modulated by SOI

In all numerical calculations, we take symmetric coupling strengths $t_{ld} = t_{rd} = 0.4$, and chemical potential $\mu_l = \mu_r$ (adiabatic spin transport). The other parameters are set $U = 5.0$ and $t_{lr} = 0.1$. Figure 2 shows the conductance G_σ versus the intradot-level position ε_d at zero and $\Phi_0/8$ magnetic flux, but with different Rashba SOI strength α_R. G_σ has the following features: (1) at zero magnetic flux, with increasing the Rashba strength α_R, the conductance curves which show typical Fano resonance are strongly modified, and the Fano parameter q which is a measure of the coupling strength between the continuum state and the resonance state [32] is made opposite [33]; (2) moreover, at zero magnetic flux it can be also found that in the presence of SOI the conductance curves of up-spin and down-spin electrons are not separated, namely, the spin polarization is nearly zero (see the figure 2(a)-(e)). These results approved the conclusions in the previous literature [22], which demonstrates our theoretical model and method should be reliable; (3) at nonzero magnetic flux (Here $\Phi = \Phi_0/8$ a weaker magnetic field is considered.), it is clearly seen that, properly modulated the SOI, the spin electrons are polarized (see the figure 2(g)-(i)); (4) when the spin polarization occurs, the Fano parameter q will be different for up-spin and down-spin electrons. As shown in Fig. 2(g), for down-spin electrons, the conductance presents two symmetric Coulomb blockade peaks located at 0 and $-U$, this implies Fano resonance disappears, while the up-spin conductance curve displays the unambiguous Fano resonance. Nevertheless, in the Fig. 2(i), the results are reversed. In addition, as shown in Fig.2 (h), for SOI parameter θ being 1.5, both the down-spin and up-spin conductance shapes display Fano resonance, whereas the Fano parameter q is inversed. The change of Fano resonance shape reflects the interference for electron transport through the two arms of AB ring. Consequently, these results indicate that when the spin-polarized electrons are transporting in the structure, the coherence of them is dramatically prominent. In contrast, in Ref. 39 the dependence of the Fano factor on the Rashba effect has been presented without considering spin polarization. To the best of our knowledge, it is the first attempt for studying the coherent behaviors of the spin-polarized electrons using the AB ring and quantum dot (QD) hybrid configuration. These results maybe indicate the direction information for the coherent behaviors of the spin-polarized electrons. (5) As $\theta = 3.0 \approx \pi$ the spin polarization is demolished, seen in Fig. (j). These results may be understood from the interference term of spin-dependent transmission probability, which is approximatively proportional to $\sim \cos(\varphi + \sigma\theta)$ [40]. For magnetic flux $\Phi = \Phi_0/8$, then $\varphi = 2\pi\Phi/\Phi_0 = \pi/4$, and $\theta = 3.0 \approx \pi$, the up-spin transmission probability is approximatively proportional to $\sim -\cos(\frac{\pi}{4})$, while down-spin transmission probability is approximatively proportional to $\sim \cos(\frac{3\pi}{4})$, that is, the orientation of up-spin Fano resonance curve is opposite to that in Fig.2 (b), but for down-spin it is the same as in Fig.2(e). Therefore, the spin polarization is almost zero for $\theta = 3.0 \approx \pi$. These results also imply that the spin polarization will vanish periodically by π with the external electric field varying. In the present technology, the above theoretical parameters can be realized experimentally. In a recent experiment [18], SOI strength α_R was successfully modulated over 2×10^{-12} eVm. Moreover, some experiments [16] had measured that α_R could reach 3×10^{-12} eVm. Such as, for $m^* = 0.036\, m_e$, if the length of the QD is the typical value 100 nm , then $\theta = \alpha_R m^* L/\hbar^2 \approx 1.5$. It means that the experimental θ is even larger and it can be also obtained, and which approves that it is possible to implement the device application from theory to experiment.

In order to further discover the properties of the phase for spin-polarized electrons with various SOI, the conductance G_σ as a function of magnetic flux for several intradot levels ε_d is plotted in Fig. 3. It is clearly seen that in the absence of SOI (1) as shown in Fig.3 (a) the phase changes by π across a resonance; (2) as shown in Fig.3 (b) two resonances almost possess the same phase behavior. These presentations demonstrate the characteristics of the phase for electrons transporting in the QD. (3) Moreover, from Fig.3 (a)-(c) it is found that the conductance is symmetrical for the magnetic flux, namely $G(\varphi) = G(-\varphi)$. This result shows that in the absence of the SOI the conductance follows the Onsager symmetry [34]. However, once the SOI is considered into and the electrons are spin polarized, the conductance will be no longer the even function of magnetic flux, that is, $G_\sigma(\varphi) \neq G_\sigma(-\varphi)$ as shown in Fig 3(d)-(i). It may owe to the contributions of different levels to even AB oscillations nearly cancel out (e.g., due to different parity of these levels) [35]. In addition, it is noticeable that the up-spin and down-spin electrons are out of phase. The phase difference between them is 2θ. The phase of wave function for up-spin electron goes beyond θ, while for down-spin electron it lags θ. The peculiarity maybe provides a new idea to detect the spin-polarized electrons.

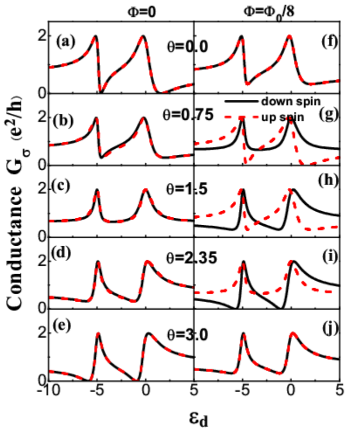

Fig.2. The conductance G_σ versus the intradot-level position ε_d at zero (right column) and $\Phi_0/8$ (left column) magnetic flux, but with different Rashba SOI strength. The red (dark gray) dashed and black solid lines correspond to up-spin and down-spin conductance, respectively. Other parameters are $t_{ld} = t_{rd} = 0.4$, $\mu_l = \mu_r$, $U = 5.0$, $t_{lr} = 0.1$, $k_B T = 0.1$.

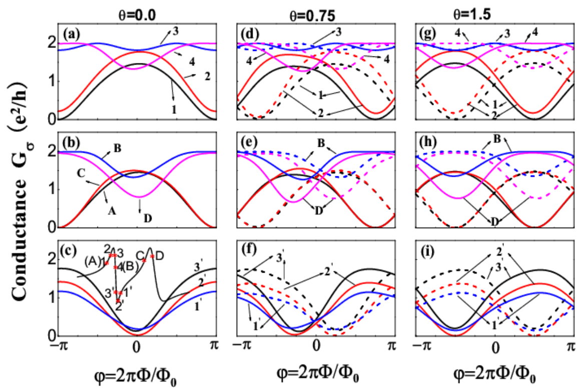

Fig.3. the conductance G_σ as a function of magnetic flux for several intradot levels ε_d with various SOI. In (a), (d) and (g) the black lines marked 1, the red (dark gray) lines marked 2, the blue (gray) lines marked 3 and the magenta (light gray) lines marked 4 are for dot level $\varepsilon_d = -6.0$, $\varepsilon_d = -5.5$, $\varepsilon_d = -5.0$ and $\varepsilon_d = -4.9$, separately. In (b), (e) and (h) the black lines marked A, the red (dark gray) lines marked B, the blue (gray) lines marked C and the magenta (light gray) lines marked D are for dot level $\varepsilon_d = -6.0$, $\varepsilon_d = -1.0$, $\varepsilon_d = -4.9$ and $\varepsilon_d = 0.5$. In (c), (f) and (i) the black lines marked 3', the red (dark gray) lines marked 2', and the blue (gray) lines marked 1' t are for dot level $\varepsilon_d = -4.7$, $\varepsilon_d = -4.5$, and $\varepsilon_d = -4.3$. The dashed and solid lines correspond to up-spin electrons and down-spin electrons, respectively. The arrow lines only use to guide the reader's eyes. Other parameters are the same as those in Fig.2. (The insert is the conductance G_σ versus the intradot-level position ε_d in the absence of magnetic flux and SOI, where the energies used in Figure3 are marked.)

B. Transmission amplitude through the QD

For the interesting phase properties of the spin-polarized electrons, in Fig. 4 we investigated the properties of the transmission amplitude through the QD for different SOI. To describe the transport properties of mesoscopic systems, the transmission amplitude $t_{\alpha\beta}$ is a very important physical quantity, which gives the correlation between the in-coming and out-going wave functions. Here the transmission amplitude tdσ transporting through the QD takes the form [36]: $t_{d\sigma} = \Gamma \int d\varepsilon \frac{\partial f(\varepsilon)}{\partial \varepsilon} G^r_{dd\sigma}(\varepsilon, \theta)$, where Γ is the effective coupling width. After some algebra, the retarded Green's function $G^r_{dd\sigma}(\varepsilon, \theta)$ can be easily obtained:

$$G^r_{dd\sigma}(\varepsilon, \theta) = \left\langle\left\langle d_\sigma \middle| d^+_\sigma \right\rangle\right\rangle^r = \frac{1 - \langle n_{\bar{\sigma}} \rangle}{\varepsilon - \varepsilon_d - \xi} + \frac{\langle n_{\bar{\sigma}} \rangle}{\varepsilon - \varepsilon_d - U - \xi} \tag{8}$$

where $\xi = -i\pi\rho(\tau_l T^*_{ld} + \tau_r T^*_{rd\sigma})/(1 + \pi^2\rho^2|t_{lr}|^2)$ ($\tau_l = T_{ld} - i\pi\rho T_{rd\sigma} T_{lr}$, $\tau_r = T_{rd\sigma} - i\pi\rho T_{ld} T_{lr}$)

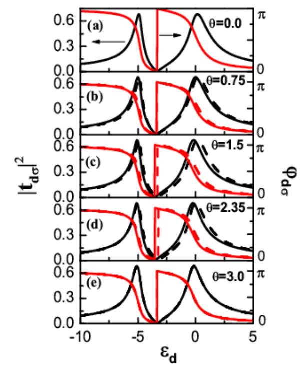

Fig.4. The magnitude (black lines) and the phase (red (dark gray) lines) (in unit of π) of transmission amplitude $t_{d\sigma}$ as a function of the single-particle energy level ε_d with several SQI at magnetic flux $\Phi = \Phi_0/8$. The dashed and solid lines correspond to up-spin electrons and down-spin electrons, respectively. The arrow lines only use to guide the reader's eyes. Other parameters are the same as those in Fig.2.

In Fig. 4, the transmission probability, i.e. the squared modulus of the transmission amplitude $|t_{d\sigma}|^2$, and the phase of the transmission amplitude $\varphi_{d\sigma} = \arg(t_{d\sigma})$ correspond to the left and right axis, respectively. The results show that the up-spin and down-spin transmission probability $|t_{d\sigma}|^2$ curves are dominated by two Coulomb peaks at $\varepsilon_d = 0$ and $\varepsilon_d = -U$. Moreover, these two peaks show a typical Fano resonance shape due to the interference of electrons passing through the two arms of AB ring, which is in agreement with previous theoretical and experimental studies [1]-[2],[8]. The transmission phase of such a Fano scatterer shows the continuously increase by π as the energy of the dot level is tuned through a resonance. It theoretically turns out that the phase rigidity is broken in the two-terminal and closed structure, which is well agree with the experimental works of Kobayashi et al. [6]. In addition, in the presence of SOI a phase lapse for up-spin or down-spin is displayed, which is expected where the Fano curve shape goes through a transmission zero [37] (seen in the figure 4(b)-(e)). The energy at this occurs depends on the Fano parameter q and will coincide with the resonance energy for $q = 0$. With increasing magnitude of q the phase jump moves away from the resonance position and occurs in the middle between resonances. The behavior of the phase lapse in company with transmission zero may still be explained from the Friedel sum rule [10], which had been demonstrated that it is valid when internal orbital and spin degrees of freedom are taken into account [38]. Moreover, according to the recent theoretical literatures [39], this phenomenon may also be accounted for the interaction-induced 'population switching' of energy levels. In contrast to the fig. 3, these results imply that although at fixed dot level, the AB oscillation phase between up-spin and down-spin electrons is shifted, the transmission phase behaviors of them are analogical across a resonance tuned by dot level.

C. Comparison with external magnetic field

Why it can give birth to the above novel phenomena to modulate the SOI? To answer the question, in Fig. 5, when an external magnetic field is added on the QD or a small quantity of magnetic flux is permeated into the QD*, the AB oscillation of conductance is plotted at different dot levels. It is found that, in spite of the up and down-spin electrons are polarized, both of them obey the Onsager symmetry, and they are in phase, i.e. phase difference is zero, or in anti-phase. The results indicate that, in contrast to figure 3, the presence of the external magnetic field on QD can induce spin polarization, but it is more difficult to distinguish between the up-spin and down-spin electrons relying on the AB phase. The reason may originate from the unlike properties of SOI and external magnetic field: When a system has spin-orbit coupling, each eigenenergy level is still at least twofold degenerate, i.e., the so-called Kramer's degeneracy is existing. Yet it gives rise to an extra phase factor $-\sigma\theta$ in the hopping matrix element between the leads and the QD, noting that this phase factor is dependent on the electronic spin σ. Which makes the two-terminal device become a three-terminal configuration, and makes the current conservation and the time-reversal symmetry destroyed. Therefore, both the up-spin and down-spin conductance do not follow the Onsager symmetry, and they are out of phase. In the presence of external magnetic field on the QD, although the spin degenerate energy levels are demolished by Zeeman Effect, the transport path of electron is not varied. Thereby, the existence of external magnetic field only induces the spin polarization; however, it can not destroy the Onsager symmetry. Moreover, these behaviors may also be understood from the different actions of electric and magnetic fields on electron transport. Transporting in our model is about the motion of two entities — charge and spin, in two fields — electric and magnetic fields. Therefore there are a total of four actions due to the fields on the charge and spin: based on magnetic field (i) the Lorentz force on the moving charge; (ii) the magnetic force on the spin (Zeeman); owing to electric field (iii) the electric or Coulomb force on the charge; and (iv) the electric force on the moving spin. Of these four actions, (i) manipulates the movement path of the charge freedom of electron, but (iv) which came from a purely relativistic effect [40] controls the motion route of the spin freedom of electron, and makes paths of up-spin and down-spin electron separated. Consequently, electric and magnetic fields bring dissimilar phase characteristics for spin-polarized electron transport through the structure. To further show the difference, the AB oscillation synchronously modulated by Rashba SO interaction and external magnetic field in Fig.6. It is obviously shown that on the effect of Rashba SO interaction the conductance will be no longer the even function of magnetic flux, that is, $G_\sigma(\varphi) \neq G_\sigma(-\varphi)$.

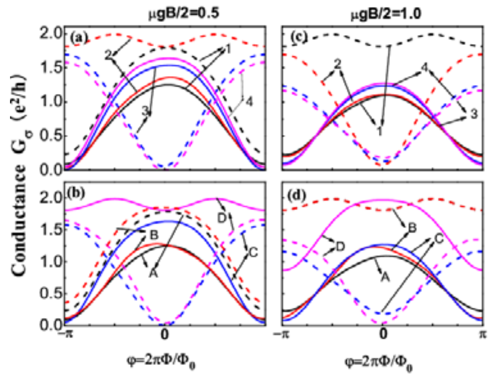

Fig. 5. The conductance G_σ as a function of magnetic flux for several intradot levels ε_d in the presence of Zeeman Effect on the QD. In (a) and (c) the black lines marked 1, the red (dark gray) lines marked 2, the blue (gray) lines marked 3 and the magenta (light gray) lines marked 4 are for dot level $\varepsilon_d = -6.0$, $\varepsilon_d = -5.5$, $\varepsilon_d = -5.0$ and $\varepsilon_d = -4.9$, separately. In (b) and (d) the black lines marked A, the red (dark gray) lines marked B, the blue (gray) lines marked C and the magenta (light gray) lines marked D are for dot level $\varepsilon_d = -6.0$, $\varepsilon_d = -1.0$, $\varepsilon_d = -4.9$ and $\varepsilon_d = 0.5$. The dashed and solid lines correspond to up-spin electrons and down-spin electrons, respectively. The arrow lines only use to guide the reader's eyes. Other parameters are the same as those in Fig.2.

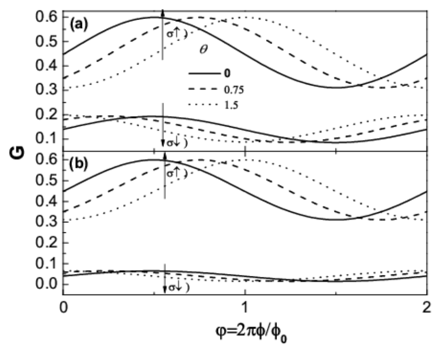

Fig. 6. The conductance vs. magnetic flux for different Rashba SO interaction in the presence of magnetic field applied on the QD. a) $\mu gB/2=1$, b) $\mu gB/2=3$. The symbols $\sigma\uparrow$ and $\sigma\downarrow$ stand for up spin and down spin respectively. The parameters used are $\varepsilon_d = 0.0$.

IV. CONCLUSIONS

In conclusion, using the powerful method-Keldysh non-equilibrium Green functions, The phase of spin-polarized electrons in a quantum dot embedded in a ring has been investigated with Rashba spin-orbit interaction. It is interestingly found that when the SOI is presented, the electrons will be spin polarized and the conductance will be no longer the even function of magnetic flux, that is, $G_\sigma(\varphi) \neq G_\sigma(-\varphi)$. In addition, it is noticeable that the up-spin and down-spin electrons are out of phase. The phase difference between them is 2θ. The phase of wave function for up-spin electron goes beyond θ, while for down-spin electron it lags θ. These results have revealed that it is possible to detect and distinguish the spin-polarized

* For InAs material $g \sim 14$, so that $B \approx 0T$ for $\mu gB/2=0$; $B \approx 1T$ for $\mu gB/2=0.5$; $B \approx 2T$ for $\mu gB/2=1$. Defining the first as $\varphi = 0$, the second and the third correspond to $\varphi = \pi/2$ and $\varphi = \pi$, respectively, as is calculated from the AB period ~2.0 T.

electrons by controlling SOI. Moreover, it theoretically approved that the phase rigidity is broken in the two-terminal and closed structure, which is well agree with the experimental works of Kobayashi et al [6].

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Interests:

1.Spintronics; Many-body stronger correlation and Kondo effect in artificial nano-system;

2.Quantum transport and electron transport of nano-system;

3.Computational modeling and simulation of nanostructured materials;