Abstract—Color visual cryptography scheme is useful in systems that need access control management and authentication like digital signature schemes used in banks that demand multi partner shares. In one of the basic visual cryptography schemes for color images, proposed by Hou et. al. it is claimed that an adversary cannot attain any knowledge about the secret image without the black mask. While by security analysis of this scheme we observe that having three color shares and stacking them, an adversary is able to distinguish color boundaries of the image and by considering the available colors of the image he can reach to the main image with a non negligible probability. In this paper, we utilize Arnold’s cat matrix to enhance the security of this scheme by means of pixel scrambling. In the proposed scheme, we mislead the adversary who stacks three color shares of the image to reconstruct the main image. Therefore, without a meaningful loss in the speed of encryption procedure, the security of Hou scheme is increased. The simulation results justify our claim.

Index Terms—Visual cryptography, halftone method, pixel scrambling

I. INTRODUCTION

Visual cryptography scheme was first introduced in 1994 by Shamir and Noar [1]. In this scheme the secret image is divided into some shares in such a way that no information can be inferred about the secret image only by one image. But when shares are stacked, the secret image will be unfolded such that the human eye can easily distinguish it. It is noteworthy that using this cryptographic method expands the secret image size by the factor of four, because in this method the secret pixel as can be seen in “Fig. 1”, is substituted by a four-pixel block. As an example if a pixel in the secret image is black, we can consider one out of six black pixels in “Fig. 1”. As an example if the fifth row is selected, two blocks shown in the fifth row will be selected for share #1 and share #2. So when these two blocks of two shares are overlaid, the black pixel will be resulted [2]. Visual cryptography schemes are also presented for color and grey level images. Visual cryptography for color images was first introduced by Hou in which he proposed three algorithms in which the first one of them contains three color shares and a black mask and the image resulted from decryption has a brightness of 50 percent while two other algorithms contain a brightness of 25 percent and two color shares [3-6]. In accordance with the foregoing security analysis, the security breach of the first algorithm is apparent. In which only two color shares out of the total three can provide adversary with some information about the secret image. In this regard, the boundaries of the image show up and the adversary can color the resulting image depending on the available colors of the image. In this paper in order to enhance the security of the predecessor scheme proposed by Hou, using the Arnold’s cat matrix, we scramble the pixels of the image in such a way that boundaries of the image are not distinguishable by stacking shares and the adversary will be mislead by observing more objects in the image. We can create a meaningful image as well such that the adversary is not able to distinguish the main image.

This paper is organized as follows, In Section II. the Hou algorithm is reviewed. Section III describes the security analysis of the Hou algorithm, In Section IV our proposed modification is presented and its security will be analyzed. In Section V, conclusions are drawn and further studies are discussed.

Fig. 1. Division of pixels in subpixels in visual cryptography

II. THE HOU VISUAL CRYPTOGRAPHY SCHEME

In Hou algorithm the main image is coded in binary form using the Halftone method and then each color is encoded in accordance with the method shown in Table I. It is noteworthy that each pixel of the halftone image corresponds to one block in each color share. As can be seen in Table I, the selection criterion of the blocks in color shares is heavily dependent on the color of the main image and also the selection of the incidental block for the mask share [3].

<table>
<thead>
<tr>
<th>Table I: The Hou Visual Cryptography Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mask</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>(0,0,0)</td>
</tr>
<tr>
<td>(1,0,0)</td>
</tr>
<tr>
<td>(0,1,1)</td>
</tr>
<tr>
<td>(0,1,0)</td>
</tr>
<tr>
<td>(1,0,1)</td>
</tr>
</tbody>
</table>

Two out of four subpixels of a block are black and the remaining subpixels are white. And also in each share two
out of four blocks that correspond to one pixel in the secret image, are white and the remaining blocks are color images and as seen in the first row of Table I, the block that corresponds with the white pixel, contains two white subpixels and two black subpixels and the density of the presence of three colors of Cyan, Magenta and Yellow in a 2 × 2 block is 1/2. As can be seen in the 8th row of table I, the 4 pixels that correspond to the black pixel are all black and the density of the presence of three colors of Cyan, Magenta and Yellow is one in each 2 × 2 block. This procedure can be analyzed for other six colors in the same way. “Fig. 2” depicts a secret image and its corresponding halftone image. “Fig. 3” also shows four encrypted shares of “Fig. 2” and also the image resulted from stacking all shares. It is noteworthy that in the proposed visual cryptographic scheme, size of the image is enlarged with a factor of four and the results of encryption are shown with a factor of 1/4.

III. SECURITY ANALYSIS OF HOU SCHEME AND PIXEL SCRAMBLING

In the Hou algorithm mentioned in Section II, the black mask is regarded as the main mask and Hou believed that no information about the secret image can be inferred without the black mask even in the presence of three other shares. In this section it is shown that the adversary is able to retrieve the main secret image with a high probability using the information of number of the colors available in the main image.

A. Security Analysis

In this subsection we introduce a Boolean matrix to calculate the probability of success for an adversary to retrieve the main secret image using only two or three shares.

Assume that the 3-tuple of \( (C, M, Y) \) with the condition of \( C, M, Y \in \{0,1\} \) shows the color of one pixel in the halftone secret image. \( M_{(C,M,Y)} \) is a 4 × 4 Boolean matrix that corresponds with four blocks of four pixels belonging to the black mask share and the remaining three other shares[7]. The first row of this matrix depicts the block that corresponds with the black mask and three other rows show the blocks belonging to the Cyan, Magenta and Yellow shares respectively. Each row contains four elements that corresponds to four pixels in each block. These values from left to right belong to the left top, right top, left bottom and right bottom pixels of the image. Two out of four values shown in each row must be set to one the remaining values must be zero. One and zero indicate the presence and absence of a color respectively [7]. In this regard, the concept of similar and complementary patterns are introduced in the sequel.

1) Similar pattern. If the status of zeros and ones are alike in two rows of a Boolean matrix, then they have a similar pattern.

2) Complementary pattern. In a Boolean matrix if the status of zeros in one row is similar with the status of ones in another row, then these rows have a complementary pattern.

Boolean matrices have these characteristics that come in the sequel [7]. If columns of a Boolean matrix are permuted, then the pattern of the blocks corresponding with the black mask and C, M and Y shares will change. But the result of the stacking these shares will not change (after applying OR function for all the four rows) for all the Boolean matrices that are generated with the permutation of the columns of \( M_{(C,M,Y)} \). Each \( M_{(C,M,Y)} \) matrix contains one pair of similar columns and it can be observed that \( E_{(C,M,Y)} \) contains 6 Boolean function for a 3-tuple of \( (C,M,Y) \).

It is significant to note that security of the secret image should not be dependent on the color formation or pattern. While in the Hou visual cryptography scheme, each two rows of the Boolean matrix have either similar patterns or complementary patterns and if the adversary is able to distinguish the boundaries evident in the stacking the three shares, can attack this scheme. In the sequel an example is shown to prove the insecurity of this algorithm. In “Fig. 4” the secret image consisting of four colors is demonstrated and it is assumed that the adversary has three color shares (without the mask depicted in “Fig. 5”). The adversary by stacking three shares, observes that the image consists of four general parts as a whole (“Fig. 5D”). “Fig. 6” shows the selected part in “Fig. 5D” with the enlargement factor of 800%.

(E) The recovered image after stacking four shares

Fig. 3. (A) The black mask, (B,C,D)The color shares, (E)The image resulting from stacking the four shares.
is composed of four different colors and there will be

Fig. 5. (A,B,C) three color shares, (D) The recovered omage with stackingof
corresponds to the matrix

eight matrices by four distinct groups. Furthermore without
latter consists of all combinations that three shares are in
the other block possesses the complementary pattern and the
corresponding blocks in two shares have similar patterns and
two colors. The former consists of instances in which the
colors and the second group contains parts with more than
different parts which are evident in the image, lie in two
can be used to interpret other parts of the image. By and large,
blocks are in complementary patterns. The same reasoning
Magenta and Yellow are in similar patterns and Yellow
part Cyan and Magenta colors are evident which show that
in the second
Cyan and Magenta are in similar patterns and the Yellow
Magenta, it can be inferred that in this section, color blocks of
Since three colors that are stacked are Cyan, Yellow and
the magnified part of “Fig. 9” , four detached divisions are
1) Matrices in which Yellow and Magenta show up with
similar patterns and Cyan comes in complementary pattern. The Boolean matrices of Red and Cyan colors
are depicted below.

\[
\text{Cyan} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
\end{bmatrix} \quad \text{Red} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 \\
\end{bmatrix}
\] (1)

2) Matrices in which Cyan and Magenta show up with
similar patterns and Yellow comes in complementary pattern with them. The Boolean matrices of Yellow and
Blue colors are depicted below.

\[
\text{Blue} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
\end{bmatrix} \quad \text{Yellow} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
\end{bmatrix}
\] (2)

3) Matrices in which Yellow and Cyan are in similar
patterns while Magenta is in complementary pattern. The Boolean matrices of Magenta and Green are
as shown below.

\[
\text{Green} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
\end{bmatrix} \quad \text{Magenta} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
\end{bmatrix}
\] (3)

4) Matrices in which all three colors of Cyan, Magenta and
Yellow are in similar patterns. The Boolean matrices of
Black and White colors are shown below.

\[
\text{Black} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 \\
\end{bmatrix} \quad \text{White} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
\end{bmatrix}
\] (4)

Based on what stated above, as an example, for the first
and second zone of “Fig. 7”, the adversary has two options
for coloring the zone, etc. Therefore, the probability of the
adversary to guess the main image is

\[
P_s = \frac{4}{4 \times 8/35} = \frac{8}{35}
\] (5)

Fig. 7. The colored image for security analysis done by an adversary

If this approach is repeated for other images with different
colors and different shares, the results shown in Table II
depicts the probability at which the adversary can conceive
the secret image.

<table>
<thead>
<tr>
<th>Number of colors of the secret image</th>
<th>Number of available shares of an adversary</th>
<th>Probability of recovering the main secret image</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>4/7=57%</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6/7=86%</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4/7=57%</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>8/35=23%</td>
</tr>
</tbody>
</table>
According to the aforementioned statements, we can bring the same reasoning for images containing more colors. In “Fig. 7” we can see that the adversary can recolor the image that is formed by stacking three color shares.

IV. ENHANCING SECURITY USING THE PROPOSED METHOD

In Section III, it was mentioned that the adversary is able to recognize the boundaries of the main image with the knowledge of three color shares and he can achieve to the main image with a large probability. In this Section it is shown that if the main image is pixel scrambled before it is encrypted, the security of this scheme will be enhanced, because the adversary using the information of the parts he has, cannot attain correct information about boundaries of the image. Pixel scrambling can be achieved using a variety of methods such as the Baker’s map, chaos, etc. [8] but in this paper it has been achieved using the Arnold’s cat algorithm [9]. Map of the Arnold’s cat algorithm can be derived using equations (6) and (7).

\[
\begin{align*}
    x_{n+1} & = A \begin{bmatrix} x_n \cr y_n \end{bmatrix} \mod N \\
    A & = \begin{bmatrix} 1 & a \\
        b & 1 + ab \end{bmatrix}
\end{align*}
\]

In these methods tuple of \((x_{n+1}, y_{n+1})\) shows the location of the pixel in the \(n + 1\)th round, \((x_n, y_n)\) depicts location of the pixel in the \(n\)th round and \(a\) and \(b\) are natural numbers.

If the image shown in “Fig. 4” is scrambled, (the image resulting from stacking three color images is depicted in “Fig. 8”) then the adversary will no longer distinguish the correct boundaries of the image with stacking the parts he already possesses. The periodicity of scrambling of the image is shown in equations (8) and (9).

\[
\begin{align*}
    r_m & = A^m r_0 \mod N \\
    r_T & = A^T r_0 \mod N = r_0
\end{align*}
\]

In Eq.9, T depicts the period i.e. after T times of scrambling using the Arnold’s cat, each pixel goes back to its first place. As can be seen in “Fig. 15”, since the size of our main image is \(64 \times 64\) pixels according to [9], our period is 64 and hence pixel scrambling does not exist.

In “Fig. 9” the selected part of “Fig. 8d” is magnified 8 times and it illustrates that this time we cannot distinguish the boundaries for this image.

In order to scramble the pixels in each step, we can send the number of pixel scrambling to the receiver using a key exchange algorithm (e.g. Diffie-Hellman key exchange) and in order to further misleading the adversary, we can choose different number of pixel scrambling rounds using the Arnold’s cat matrix for each color share and the adversary is intrigued because he thinks that the image contains multi objects.

V. CONCLUSION

The first proposed algorithm for visual cryptography proposed by Hou featured the brightness of 50 percent which seemed fair comparing to the other algorithms while by
analyzing the security of this scheme, we found out that the adversary can disclose the boundaries of the main image without share of the black mask and he can then color the image depending on the number of colors of the main image and achieve to the main image with a non-negligible probability. Using the Arnold’s cat matrix, we scramble the pixels of the image in such a way that boundaries of the secret image cannot be distinguished by stacking shares and the adversary will be intrigued by observing more objects in the stacked image. The results of simulation depict that we can create meaningful images using different number of rounds for selected images (in other terms extended visual cryptography scheme can be utilized). In order to further intriguing the adversary, we can first create shares of Visual cryptography and then scramble each share in various rounds using the Arnold’s cat matrix and then we should exchange the scrambling schemes for each share using key exchange methods such as Diffie-Hellman key exchange method.

ACKNOWLEDGMENT

The authors are thankful of the financial support provided by Iran Telecommunication Research Center (ITRC). Also, The authors also thank Mr Arash Karimi for his helps in reviewing the early draft of the paper.

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