New Guaranteed H_∞ Performance State Estimation for Delayed Neural Networks

Won Il Lee and PooGyeon Park

***Abstract*—In this paper, a new guaranteed performance state estimation problem for static neural networks with time-varying delay is investigated. A new Lyapunov-Krasovskii functional is introduced to improve the performance. Moreover, with the help of lower bound lemma, an upper-bound of a linear combination of positive functions weighted by the inverses of convex parameters is obtained. Two simulation examples are given to prove the effectiveness of the proposed theorem.**

***Index Terms*—State estimation, static neural networks, H-infinite performance, reciprocally convex approach, Time-varying delay.**

I. Introduction

Neural networks have attracted considerable attention from academic research and industrial applications during the past decades. Various successful applications have been founded in many fields such as pattern recognition, image processing, optimization problems, and adaptive control. Time delay is widely exists in many practical systems such as chemical or process control systems and networked control systems [1]. Also, time delay may exist in neural networks because of their finite switching speeds and communication time. Since these time-delay may induce system instability and performance degradation, the stability analysis of delayed neural networks has become an important issue, and many results have been reported in the literature [2]-[4].

State estimation problem of neural network is very practical and theoretically important issue, which has been studied in recent years [5]-[7]. In many practical applications, the neuron states are not always measurable in the neural networks outputs since it may be very difficult and expensive to acquire all the state information of the neuron states in large-scale neural networks. But the state information may be certainly necessary for some applications such as system modeling and state feedback control. Therefore, in this case, the neuron states should be estimated by measurements, it proves the importance of the state estimation problem for neural networks. Recently, [5] proposed a guaranteed performance state estimator for static neural networks with time-varying delay. But in the process of deriving lower bounds of one integral term, [5] introduced an approximation leading to a little conservativeness.

In this paper, we propose a new guaranteed performance state estimator for delayed neural networks based on a new Lyapunov-Krasovskii functional. By applying [8]'s lower bound lemma, an improved performance is obtained.

This paper is organized as follows. The state estimation problem is formulated in Section 2. Section 3 proposes a new guaranteed H-infinity performance state estimator for delayed static neural networks. In Section 4, two simulation examples are given to prove the effectiveness of the proposed theorem.

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II. Problem Formulation

Consider the delayed static neural network subject to noise disturbances:

$$\Sigma_N \ : \dot{x}(t) = -Ax(t) + f\,(Wx(t-d(t)) + J) + B_1 w(t), \quad (1)$$

$$y(t) = Cx(t) + Dx(t-d(t)) + B_2 w(t), \quad (2)$$

$$z(t) = Hx(t), \quad (3)$$

$$x(s) = \phi(s), \quad s \in [-\tau, 0], \quad (4)$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \mathbf{R}^n$ is the neuron state vector; $y(t) \in \mathbf{R}^m$ is the network output measurement; $z(t) \in \mathbf{R}^p$ is a linear combination of the states to be estimated; $w(t) \in \mathbf{R}^q$ is a noise disturbance belonging to $L_2[0,\infty)$; $A = diag\{a_1, a_2, \cdots, a_n\}$ is a diagonal matrix with positive entries $a_i > 0$; $W^{n\times n}$ is the interconnection matrices representing the weighting coefficients of the neurons; $f(x(t)) = [\, f_1(x_1(t), f_2(x_2(t)), \cdots, f_n(x_n(t))]^T$ is the neuron activation function; $J = [J_1, J_2, \cdots, J_n]^T$ is an external input vector; $\phi(s)$ is an initial condition on $[-\tau, 0]$, and $d(t)$ is a time-varying delay of the system satisfying

$$0 \le d(t) \le \tau, \ \ 0 \le \dot{d}(t) \le \mu \,. \quad (5)$$

In this paper, we choose a state estimator for the neural network (Σ_N) as

$$\Sigma_S \ : \dot{\hat{x}}(t) = -A\hat{x}(t) + f(W\hat{x}(t-d(t)) + J) + K[y(t) - C\hat{x}(t) - D\hat{x}(t-d(t))], \quad (6)$$

$$\hat{z}(t) = H\hat{x}(t), \quad (7)$$

$$\hat{x}(s) = 0, \quad s \in [-\tau, 0], \quad (8)$$

where $\hat{x}(t) \in \mathbf{R}^n$, $\hat{z}(t) \in \mathbf{R}^p$, and K is the state estimator gain matrix to be determined.

Define the errors to be $e(t) = x(t) - \hat{x}(t)$ and $\overline{z}(t) = z(t) - \hat{z}(t)$. Then the error-state system is represented by

$$\Sigma_E \ : \dot{e}(t) = -(A + KC)e(t) - KDe(t-d(t)) + \psi(We(t-d(t)), \hat{x}(t-d(t))) + (B_1 - KB_2)w(t), \quad (9)$$

$$\overline{z}(t) = He(t) \quad (10)$$

where $\psi(We(t), \hat{x}(t)) = f(Wx(t) + J) - f(W\hat{x}(t) + J)$.

Definition 1 : The error system (Σ_E) is said to be globally stable with H_∞ performance γ if, for some scalar $\gamma > 0$, there exists a proper state estimator (Σ_S) such that the equilibrium point of the resulting error system (9) with $w(t) \equiv 0$ is globally asymptotically stable, and

$$\left\|\overline{z}(t)\right\|_2 < \gamma \left\|w(t)\right\|_2 \quad (11)$$

Under zero-initial conditions for all nonzero $w(t) \in L_2[0, \infty)$, where $\left\|\eta(t)\right\|_2 = \sqrt{\int_0^\infty \eta^T(t)\eta(t)dt}$.

Assumption 1. The neuron activation functions in (1), $f_i(\cdot)$, satisfy the following Lipschitz condition

$$0 \le \frac{f_i(x) - f_i(y)}{x - y} \le l_i, \quad x \ne y \in \mathbf{R}, \quad (\text{i}=1,2,\cdots,n) \quad (12)$$

with $L = diag(l_1, l_2, \cdots, l_n) > 0$.

III. Guaranteed H_∞ Performance State Estimator

This section is dedicated to the design of a guaranteed H_∞ performance state estimator for the delayed static neural network. A delay dependent LMI based condition will be established.

Theorem 1. Under Assumption 1, given positive scalars τ, μ, and prescribed constant $\gamma > 0$, state estimation problem of the delayed static neural network (Σ_N) with guaranteed H_∞ performance is solvable if there exist appropriately dimensioned matrices $P > 0$, $Q_i > 0$ (i=1,2,3), $S_j > 0$, $R_j > 0$ (j=1,2), Z and two diagonal matrices $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_n) > 0$, $\Gamma = diag(\gamma_1, \gamma_2, \cdots, \gamma_n) > 0$ such that the following LMIs hold

$$\begin{bmatrix} S_2 + R_1 & S \\ S & S_2 + R_2 \end{bmatrix} > 0, \quad (13)$$

$$\begin{bmatrix} \overline{\Omega}_{11} & \overline{\Omega}_{12} & S & 0 & 2R_1 & 0 & W^T L\Lambda & P & \overline{\Omega}_{19} & \overline{\Omega}_{110} \\ * & \Omega_{22} & S_2 - S & 0 & 2R_2 & 2R_1 & 0 & W^T L\Gamma & 0 & -\tau D^T G^T \\ * & * & -Q_2 - S_2 - 2R_2 & 0 & 0 & 2R_2 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -2R_1 - 2R_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -2\Lambda + Z & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & 0 & \tau P \\ * & * & * & * & * & * & * & * & -\gamma^2 I & \overline{\Omega}_{910} \\ * & * & * & * & * & * & * & * & * & -2P + X \end{bmatrix} < 0, \quad (14)$$

$$\begin{bmatrix} \overline{\Omega}_{11} & \overline{\Omega}_{12} & S & 0 & 2R_1 & 0 & W^T L\Lambda & P & \overline{\Omega}_{19} & \overline{\Omega}_{110} \\ * & \Omega_{22} & S_2 - S & 0 & 2R_2 & 2R_1 & 0 & W^T L\Gamma & 0 & -\tau D^T G^T \\ * & * & -Q_2 - S_2 - 2R_2 & 0 & 0 & 2R_2 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -2R_1 - 2R_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -2\Lambda + Z & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & 0 & \tau P \\ * & * & * & * & * & * & * & * & -\gamma^2 I & \overline{\Omega}_{910} \\ * & * & * & * & * & * & * & * & * & -2P + X \end{bmatrix} < 0, \quad (15)$$

where

$$\overline{\Omega}_{11} = -PA - A^T P - GC - C^T G^T + Q_1 + Q_2 - S_2 + H^T H + \tau S_1 - 2R_1,$$

$$\overline{\Omega}_{12} = -GD + S_2 - S, \ \overline{\Omega}_{19} = PB_1 - GB_2,$$

$$\overline{\Omega}_{110} = -\tau A^T P - \tau C^T G^T,$$

$$\Omega_{22} = -(1-\mu)Q_1 - 2S_2 + S + S^T - 2R_2 - 2R_1,$$

$$\Omega_{55} = \Omega_{66} = -2R_1 - 2R_2 - \tau S_1, \ \Omega_{88} = -(1-\mu)Z - 2\Gamma,$$

$$\overline{\Omega}_{910} = \tau B_1^T P - \tau B_2^T G^T.$$

In this case, a desired the state estimator gain matrix K is given as $K = P^{-1}G$.

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$V(e(t)) = e^T(t)Pe(t) + \int_{t-d(t)}^t e^T(s)Q_1 e(s)ds + \int_{t-\tau}^t e^T(s)Q_2 e(s) + \int_{t-\tau}^t \dot{e}^T(s)Q_3\dot{e}(s)ds + \int_{-\tau}^0 \int_{t+\theta}^t e^T(s)S_1 e(s)dsd\theta + \tau\int_{-\tau}^0 \int_{t+\theta}^t \dot{e}^T(s)S_2\dot{e}(s)dsd\theta + \int_{-\tau}^0 \int_\eta^0 \int_{t+\theta}^t \dot{e}^T(s)R_1\dot{e}(s)dsd\theta d\eta + \int_{-\tau}^0 \int_{-\tau}^\eta \int_{t+\theta}^t \dot{e}^T(s)R_2\dot{e}(s)dsd\theta d\eta + \int_{t-d(t)}^t \psi^T(We(s), \hat{x}(s))Z\psi(We(s), \hat{x}(s))ds. \quad (16)$$

Calculating the time-derivative of $V(e(t))$ along the trajectories of system (1) and noting that $d(t)$ satisfies (5), it yields

$$\begin{aligned}\dot{V}(e(t)) \le\ & e^T(t)[-P(A+KC)-(A+KC)^T P+Q_1+Q_2+\tau S_1]e(t) \\ & -2e^T(t)PKDe(t-d(t))+2e^T(t)P\psi(We(t-d(t)),\hat{x}(t-d(t)) \\ & +2e^T(t)P(B_1-KB_2)w(t)-(1-\mu)e^T(t-d(t))Q_2e(t-d(t)) \\ & +\psi^T(We(t),\hat{x}(t))Z\psi(We(t),\hat{x}(t))-(1-\mu)\psi^T(We(t-d(t)), \\ & \hat{x}(t-d(t)))Z\psi(We(t-d(t)),\hat{x}(t-d(t)))-e^T(t-\tau)Q_2e(t-\tau) \\ & -\dot{e}^T(t-\tau)Q_3\dot{e}(t-\tau)+\dot{e}^T(t)[S_2+Q_3+\frac{\tau^2}{2}(R_1+R_2)]\dot{e}(t) \\ & -\int_{t-\tau}^{t} e^T(s)S_1e(s)ds-\tau\int_{t-\tau}^{t}\dot{e}^T(s)S_2\dot{e}(s)ds \\ & -\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)R_1\dot{e}(s)dsd\theta-\int_{-\tau}^{0}\int_{t-\tau}^{t+\theta}\dot{e}^T(s)R_2\dot{e}(s)dsd\theta.\end{aligned} \tag{17}$$

Using Jensen's inequality [9], one can obtain

$$\begin{aligned} & -\int_{t-\tau}^{t} e^T(s)S_1e(s)ds \\ & =-\int_{t-\tau}^{t-d(t)} e^T(s)S_1e(s)ds-\int_{t-d(t)}^{t} e^T(s)S_1e(s)ds \\ & \le -(\tau-d(t))[\frac{1}{\tau-d(t)}\int_{t-\tau}^{t-d(t)} e^T(s)ds]S_1[\frac{1}{\tau-d(t)}\int_{t-\tau}^{t-d(t)} e(s)ds] \\ & \quad -d(t)[\frac{1}{d(t)}\int_{t-d(t)}^{t} e^T(s)ds]S_1[\frac{1}{d(t)}\int_{t-d(t)}^{t} e(s)ds],\end{aligned} \tag{18}$$

$$\begin{aligned} & -\tau\int_{t-\tau}^{t}\dot{e}^T(s)S_2\dot{e}(s)ds \\ & =-\tau\int_{t-\tau}^{t-d(t)}\dot{e}^T(s)S_2\dot{e}(s)ds-\tau\int_{t-d(t)}^{t}\dot{e}^T(s)S_2\dot{e}(s)ds \\ & \le -\frac{\tau}{\tau-d(t)}[e^T(t-d(t))-e^T(t-\tau)]S_2[e(t-d(t))-e(t-\tau)] \\ & \quad -\frac{\tau}{d(t)}[e^T(t)-e^T(t-d(t))]S_2[e(t)-e(t-d(t)],\end{aligned} \tag{19}$$

$$\begin{aligned} & -\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)R_1\dot{e}(s)dsd\theta \\ & =-\int_{-d(t)}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)R_1\dot{e}(s)dsd\theta-\int_{-\tau}^{-d(t)}\int_{t+\theta}^{t-d(t)}\dot{e}^T(s)R_1\dot{e}(s)dsd\theta \\ & \quad -\int_{-\tau}^{-d(t)}\int_{t-d(t)}^{t}\dot{e}^T(s)R_1\dot{e}(s)dsd\theta \\ & \le -2[e^T(t)-\frac{1}{d(t)}\int_{t-d(t)}^{t} e^T(s)ds]R_1[e(t)-\frac{1}{d(t)}\int_{t-d(t)}^{t} e(s)ds] \\ & \quad -2[e^T(t-d(t))-\frac{1}{\tau-d(t)}\int_{t-\tau}^{t-d(t)} e^T(s)ds]R_1[e(t-d(t)) \\ & \quad -\frac{1}{\tau-d(t)}\int_{t-\tau}^{t-d(t)} e(s)ds]+[e^T(t)-e^T(t-d(t))]R_1[e(t) \\ & \quad -e(t-d(t))]-\frac{\tau}{d(t)}[e^T(t)-e^T(t-d(t))]R_1[e(t)-e(t-d(t))],\end{aligned} \tag{20}$$

$$\begin{aligned} & -\int_{-\tau}^{0}\int_{t-\tau}^{t+\theta}\dot{e}^T(s)R_2\dot{e}(s)dsd\theta \\ & =-\int_{-d(t)}^{0}\int_{t-d(t)}^{t+\theta}\dot{e}^T(s)R_2\dot{e}(s)dsd\theta-\int_{-\tau}^{-d(t)}\int_{t-\tau}^{t+\theta}\dot{e}^T(s)R_2\dot{e}(s)dsd\theta \\ & \quad -\int_{-d(t)}^{0}\int_{t-\tau}^{t-d(t)}\dot{e}^T(s)R_2\dot{e}(s)dsd\theta \\ & \le -2[\frac{1}{d(t)}\int_{t-d(t)}^{t} e^T(s)ds-e^T(t-d(t))]R_2[\frac{1}{d(t)}\int_{t-d(t)}^{t} e(s)ds \\ & \quad -e(t-d(t))]-2[\frac{1}{\tau-d(t)}\int_{t-\tau}^{t-d(t)} e^T(s)ds-e^T(t-\tau)] \\ & \quad \times R_2[\frac{1}{\tau-d(t)}\int_{t-\tau}^{t-d(t)} e(s)ds-e(t-\tau)] \\ & \quad +[e^T(t-d(t))-e^T(t-\tau)]R_2[e(t-d(t))-e(t-\tau)] \\ & \quad -\frac{\tau}{\tau-d(t)}[e^T(t-d(t))-e^T(t-\tau)]R_2[e(t-d(t))-e(t-\tau)].\end{aligned} \tag{21}$$

Let us define $\alpha = d(t)/\tau$ and

$$\lambda(t)=col\{\sqrt{\frac{1-\alpha}{\alpha}}(e(t)-e(t-d(t)),\sqrt{\frac{\alpha}{1-\alpha}}(e(t-d(t))-e(t-\tau))\}.$$

Then, gathering the positive functions weighted by the inverses of the convex parameters $\{\alpha, 1-\alpha\}$, or equivalently $\{\tau/d(t), \tau/(\tau-d(t))\}$ from (19)-(21) and applying lower bound lemma [8] for S satisfying (13), we have

$$\lambda^T(t)\begin{bmatrix} S_2+R_1 & S \\ S^T & S_2+R_2\end{bmatrix}\lambda(t) \ \ge\ 0$$

which produces an upper-bound as

$$\begin{aligned} & -\frac{\tau}{d(t)}[e^T(t)-e^T(t-d(t))](S_2+R_1)[e(t)-e(t-d(t))] \\ & -\frac{\tau}{\tau-d(t)}[e^T(t-d(t))-e^T(t-\tau)](S_2+R_2)[e(t-d(t))-e(t-\tau)] \\ & \le -\begin{bmatrix} e(t)-e(t-d(t)) \\ e(t-d(t))-e(t-\tau)\end{bmatrix}^T\begin{bmatrix} S_2+R_1 & S \\ S^T & S_2+R_2\end{bmatrix}\begin{bmatrix} e(t)-e(t-d(t)) \\ e(t-d(t))-e(t-\tau)\end{bmatrix}.\end{aligned} \tag{22}$$

By assumption 1, one can obtain that, for any $W_ie \neq 0$,

$$0 \le \frac{\psi_i(W_ie,\hat{x})}{W_ie}=\frac{f_i(W_ix+J_i)-f_i(W_i\hat{x}+J_i)}{W_ix-W_i\hat{x}} \le l_i\,,$$

where $W_i=[w_{i1}, w_{i2}, \cdots, w_{in}]$ is the i-th row vector of W. Then the following inequalities hold,

$$\begin{aligned} 0 \le & -2\psi^T(We(t),\hat{x}(t))\Lambda\psi(We(t),\hat{x}(t)) \\ & +2\psi^T(We(t),\hat{x}(t))\Lambda LWe(t),\end{aligned} \tag{23}$$

$$
\begin{aligned}
0 \le & -2\psi^T(We(t-d(t)),\hat{x}(t-d(t)))\Gamma\psi(We(t-d(t)),\hat{x}(t-d(t))) \\
& +2\psi^T(We(t-d(t)),\hat{x}(t-d(t)))\Gamma L We(t-d(t)),
\end{aligned} \tag{24}
$$

For any diagonal matrices $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_n) > 0$ and $\Gamma = diag(\gamma_1, \gamma_2, \cdots, \gamma_n) > 0$.

Define $J(t) = \int_0^t [\overline{z}^T(s)\overline{z}(s) - \gamma^2 w^T(s)w(s)]ds$ for $t > 0$. Then, for any non-zero $w(t) \in L_2[0,\infty)$,

$$
\begin{aligned}
J(t) &\le \int_0^t [\overline{z}^T(s)\overline{z}(s) - \gamma^2 w^T(s)w(s)]ds + Ve(t)) - V(e(0)) \\
&= \int_0^t [\overline{z}^T(s)\overline{z}(s) - \gamma^2 w^T(s)w(s) + \dot{V}(e(s))]ds.
\end{aligned} \tag{25}
$$

From the condition (10) and (17)- (24), it can be seen that

$$
\begin{aligned}
&\overline{z}^T(t)\overline{z}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(e(t)) \\
&\le \xi^T(t)[\Omega_1 + \tau^2\Omega_2{}^T X\Omega_2 - d(t)\Omega_3{}^T S_1\Omega_3 - (\tau - d(t))\Omega_4{}^T S_1\Omega_4]\xi(t),
\end{aligned} \tag{26}
$$

where

$$
\begin{aligned}
\xi(t) = [&e^T(t), e^T(t-d(t)), e^T(t-\tau), \dot{e}^T(t-\tau), \\
&\frac{1}{d(t)}\int_{t-d(t)}^t e(s)ds, \frac{1}{\tau - d(t)}\int_{t-\tau}^{t-d(t)} e(s)ds, \psi^T(We(t),\hat{x}(t)), \\
&\psi^T(We(t-d(t)),\hat{x}(t-d(t))), w^T(t)]^T,
\end{aligned} \tag{27}
$$

$$
\Omega_1 = \begin{bmatrix}
\Omega_{11} & \Omega_{12} & S & 0 & 2R_1 & 0 & W^T L\Lambda & P & \Omega_{19} \\
* & \Omega_{22} & S_2 - S & 0 & 2R_2 & 2R_1 & 0 & W^T L\Gamma & 0 \\
* & * & -Q_2 - S_2 - 2R_2 & 0 & 0 & 2R_2 & 0 & 0 & 0 \\
* & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -2R_1 - 2R_2 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -2R_1 - 2R_2 & 0 & 0 & 0 \\
* & * & * & * & * & * & -2\Lambda + Z & 0 & 0 \\
* & * & * & * & * & * & * & \Omega_{88} & 0 \\
* & * & * & * & * & * & * & * & -\gamma^2 I
\end{bmatrix},
$$

$$
\begin{aligned}
\Omega_{11} &= -PA - A^T P - PKC - C^T K^T P + Q_1 + Q_2 - S_2 + H^T H \\
&\quad + \tau S_1 - 2R_1, \\
\Omega_{12} &= -PKD + S_2 - S, \quad \Omega_{19} = PB_1 - PKB_2, \\
\Omega_2 &= \left[-(A+KC)\ -KD\ 0\ 0\ 0\ 0\ 0\ I\ B_1 - KB_2\right], \\
X &= S_2 + \frac{1}{\tau^2}Q_3 + \frac{1}{2}R_1 + \frac{1}{2}R_2, \\
\Omega_3 &= \left[0\ 0\ 0\ 0\ I\ 0\ 0\ 0\ 0\right],\ \Omega_4 = \left[0\ 0\ 0\ 0\ 0\ I\ 0\ 0\ 0\right].
\end{aligned} \tag{28}
$$

Since $-d(t)\Omega_3{}^T S_1\Omega_3 - (\tau - d(t))\Omega_4{}^T S_1\Omega_4$ is a convex combination of the matrices $\Omega_3{}^T S_1\Omega_3$ and $\Omega_4{}^T S_1\Omega_4$ on $d(t)$, it can be non-conservatively handled by two boundary LMIs: one for $d(t) = 0$ and the other for $d(t) = \tau$. Pre- and post-multiplying two LMIs by $diag\{I, I, I, I, I, I, I, I, PX^{-1}\}$. Using following inequality for any real $P > 0$ and $X > 0$

$$
\begin{aligned}
&PX^{-1}P - 2P + X = (P - X)X^{-1}(P - X) \ge 0, \\
&-PX^{-1}P \le -2P + X,
\end{aligned} \tag{29}
$$

And applying the change of variable such that $K = P^{-1}G$, one can deduce the LMIs (13)-(15) imply $\overline{z}^T(s)\overline{z}(s) - \gamma^2 w^T(s) \quad \times w(s) + \dot{V}(e(s)) < 0$ for $w(t) \neq 0$. Therefore, $J(t) < 0$ from (25) for $t > 0$, thus (11) holds.

Globally asymptotically stability of the equilibrium point of the error system (9) with $w(t) \equiv 0$ is achieved if $\dot{V}(e(t)) < 0$ holds. One can easily prove that the condition $\dot{V}(e(t)) < 0$ is guaranteed by the LMIs (13)-(15). We skip the specific proof due to space limitation.

IV. Simulation Examples

Two simulation examples are given in this section to illustrate the effectiveness of the developed approach.

Example 1. Consider a delayed static neural network (Σ_N) with the following parameters:

$$
A = \begin{bmatrix} 0.96 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1.48 \end{bmatrix},\ W = \begin{bmatrix} 0.5 & 0.3 & -0.36 \\ 0.1 & 0.12 & 0.5 \\ -0.42 & 0.78 & 0.9 \end{bmatrix},
$$

$$
B = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix},\ B_2 = -0.1,\ C = \begin{bmatrix} 1 & 0 & -2 \end{bmatrix},
$$

$$
D = \begin{bmatrix} 0.5 & 0 & -1 \end{bmatrix},\ H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix},\ J = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
$$

The activation function, the time varying delay, and the noise disturbance are taken as $f(x) = \tanh(x)$ with $L = I$, $d(t) = 0.5 + 0.5\cos(2.4t)$ with $\tau = 1$ and $\mu = 1.2$, and $w(t) = 1/(0.8 + 1.2t)$ for $t > 0$, respectively.

Then, solving theorem 1 by resorting to the LMI solver in the Matlab LMI Control Toolbox, the state estimator gain matrix can be found as

$$
K = \begin{bmatrix} 0.0728 \\ -0.1987 \\ -0.2965 \end{bmatrix},
$$

With the optimal H_∞ performance index $\gamma_{min} = 1.3705$. It is easy to notice that this result is an improved result than $\gamma_{min} = 1.6002$ in [5]. Fig. 1 represents the error $e(t)$ for 10 random initial values.

Example 2. Consider a delayed static neural network (Σ_N) with the following parameters:

$$
A = \begin{bmatrix} 1.56 & 0 & 0 \\ 0 & 2.42 & 0 \\ 0 & 0 & 1.88 \end{bmatrix},\ W = \begin{bmatrix} -0.3 & 0.8 & -1.36 \\ 1.1 & 0.4 & -0.5 \\ 0.42 & 0 & -0.95 \end{bmatrix},
$$

$$B = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix},\ B_2 = 0.4,\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

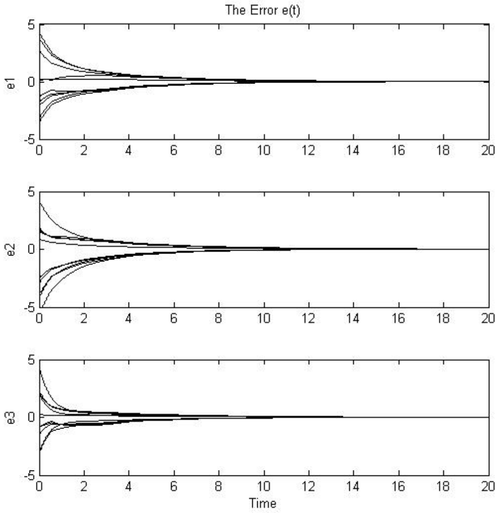

Fig. 1. The error $e(t)$ for 10 random initial value.

TABLE I: COMPARISON OF THE OPTIMAL H_∞ PERFORMANCE INDEX γ

(τ, μ)	(0.9, 0.3)	(0.9, 0.5)	(0.8, 0.7)
[5]	0.3404	0.3871	0.2883
Theorem 1	0.3247	0.3689	0.2430

$$D = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix},\ H = \begin{bmatrix} 1 & 0 & 0.5 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix},\ L = I\ .$$

Using Theorem 1, the optimal H_∞ performance index γ_{min} can be derived at different (τ, μ), and these are summarized in Table 1. It is easy to see that our method can obtain much better γ_{min} than those in [5].

V. CONCLUSION

In this paper, the guaranteed H-infinite performance state estimation problems is studied for delayed static neural networks. Based on a new Lyapunov-Krasovskii functional, we solved the guaranteed H-infinite performance state estimation problem. Moreover, with the help of [8]'s lower bound lemma, we could obtain an improved H-infinite performance results for delayed static neural networks. It is shown that the guaranteed H-infinite performance state estimator gain matrix can be found by solving LMIs. Two simulation examples proved the improvement of the proposed theorem compared to existing one. It is worth noticing that the proposed theorem can be widely applicable in control fields such as state feedback control problems and large scale neural networks, and so on.

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