Synchronization Between Two Different Chaotic Systems Using Adaptive Sliding Mode Control

Zahra Rabiei, G. Reza.Bidari, and Naser Pariz

***Abstract*—This paper presents chaos synchronization between two different chaotic systems via a continuous sliding mode control with integral action. An adaptive technique has been used to estimate the controller gain. Synchronization by means of only one controller is discussed. This method is applied to achieve chaos synchronization for each pair of dynamical systems Lorenz, Chen and Lu, but only a part of the simulation results are presented in this paper. The proposed scheme is then applied to a secure communication system. Simulation results verify the proposed scheme is success in the communication application.**

***Index Terms*—Chaos synchronization, chen system, lorenz system, lu system, sliding mode control.**

I. INTRODUCTION

Since Pecora and Carroll introduced a method [1] to synchronize chaotic systems and realized it in electronic circuits, chaos synchronization has attracted the interest of many researchers as a key technique of secure communication and various effective control methods such as, adaptive variable structure control [2], backstepping control [3], H^{∞} control method [4] have been proposed to achieve chaos synchronization. Chaotic system is a very complex dynamical nonlinear system and its response possesses some characteristics, such as extensive sensitivity to initial conditions, broad spectrum of Fourier transform and fractal properties of the motion in phase space. In 1963, Lorenz found the first canonical chaotic attractor [5]. In 1999, Chen found another similar but topologically not equivalent chaotic attractor [6], as the dual of the Lorenz system. Then the duality is in the sense defined by Vanecek and Celikovsky [7] for the linear part of the system, $A=[a_{ij}]_3$ the Lorenz system satisfies the condition $a_{12}a_{21} > 0$ while the Chen system satisfies $a_{12}a_{21} < 0$. In 2002, Lu found a new chaotic system, which satisfies the condition $a_{12}a_{21} = 0$ [8]. In this paper, we apply sliding mode control to synchronize two different chaotic systems. We demonstrate this technique by Lorenz, Chen and Lu systems. Compared with the existing results in the literatures, there are two advantages which make this approach attractive. First, most of the methods need more than one variable information of the master system; but we will design a feedback controller which needs only one variable information of the master system. Second, it needs only one controller to realize synchronization between two different chaotic systems and it is easy to implement. The problem is to design a synchronization algorithm to guarantee robust global stability and force the states of the slave system to exponentially synchronize with the states of the master system, i.e., to achieve

$$\lim_{t \to \infty} \|x_m(t) - x_s(t)\| \to 0 \tag{1}$$

where $\|.\|$ is the Euclidean norm of vector, $X_m \in R^n$ and $X_s \in R^n$ are the state vectors of the master and the slave systems, respectively. In the next section the proposed controller design is presented. In section 3, the problem is described. After that, Synchronization between two different chaotic systems and simulation results are presented.

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The authors are with Islamic Azad University of Aliabad Katoul, Iran (e-mail: Zahra_rabiei@yahoo.com)

II. PROPOSED CONTROLLER DESIGN

In order to design a synchronization to achieve objective (1), adaptive sliding mode control with integral action is employed. To describe the new design and analysis, this assumptions is needed, firstly, only the master system output $y = Cx_m$ is available for feedback, and secondly, the chaotic nonlinear systems are minimum-phase. The first assumption is realistic because in most cases only one state is available for feedback from the master as well as the slave circuit. For instance, in the secure communication case only the transmitted signal y_m and receiver signal, y_s are available for feedback from measurements. Second assumption, i.e., minimum-phase implies that the zero dynamics of chaotic system converges to an attractor. If the chaotic system is transformed into the canonical form, using asymptotical stability of internal dynamics can be shown that chaotic systems are minimum-phase [3]. The first step in the sliding mode design is to specify a sliding surface on which sliding motion occurs. In the absence of integral action, we define the sliding surface s=0 by

$$s = \sum_{j=1}^{\rho-1} k_j e_j + e_\rho \tag{2}$$

where ρ is the relative degree of the chaotic nonlinear system (i.e., the lowest order time derivative such that the control command u is directly related to the output y), and the positive constants $k_1,..., k_{\rho-1}$ are chosen such that the polynomial $\lambda^{\rho-1} + k_{\rho-1} + ... + k_1$ is Hurwitz, which guarantees that when motion is constrained to the surface s=0, the tracking errors converge to zero. In ideal sliding mode control design, in order to obtain smaller errors, it is necessary to μ make smaller, which in turn, leads to

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chattering. In order to design a continuous sliding mode control with integral action [9], sliding surface s is defined as

$$s = k_0\sigma + \sum_{j=1}^{\rho-1} k_j e_j + e_\rho . \quad (3)$$

where $k_0 > 0$, is arbitrary, $k_1,..., k_{\rho-1}$ is retained from the ideal sliding mode control design, and σ is the output of

$$\dot{\sigma} = -k_0\sigma + \mu sat(s/\mu), \sigma(0) \in [-\mu/k_0, \mu/k_0] \quad (4)$$

where sat is the standard saturation function.and μ is a small positive parameter to be specified. As mentioned above, only the output error e_1 is accessible and the others states error must be estimated, thus a high-gain observer (HGO) is employed

$$\begin{cases} \dot{\hat{e}} = \hat{e}_{j+1} + \alpha_j (e_1 - \hat{e}_1)\varepsilon^j, 1 \le j \le \rho \\ \dot{\hat{e}}_\rho = \alpha_\rho (e_1 - \hat{e}_1)\varepsilon^\rho, \end{cases} \quad (5)$$

where ε> 0 is a design parameter, and the positive constants α_j are chosen such that the roots of $\lambda^\rho + \alpha_1\lambda^{\rho-1} + ... + \alpha_{\rho-1}\lambda + \alpha_\rho = 0$ have negative real parts. Finite-time convergence to, and invariance of, s=0 can be achieved by choosing $u = u_{eq} + u_{sw}$, where u_{eq} is the equivalent control and u_{sw} is the switching control. Since our design requires the control be bounded, a possible simplification of the controller is to choose the equivalent control to be zero and coefficient of the switching component to be constant, i.e.

$$u = -Ksat(\hat{s}/\mu) \quad (6)$$

K is a positive constant value such that is dependent on type of system. Continuous sliding mode control can eliminate chattering but often at the cost of a nonzero steady-state error, which is proportional to μ. In order to obtain smaller errors, it is therefore necessary to make μ smaller, which in turn, leads to chattering again. So the integral action is defined. But now we do not need μ to be arbitrarily small; we only need it to be "small enough". Since the upper bound *K* of different systems is not easy to determine, therefore we use an adaptive gain [2] for controller as follows

$$\begin{aligned} K &= K_0 + \hat{K}_1 \\ k_0 &= \hat{K}_1 \end{aligned} \quad (7)$$

where K_0 is a positive constant value and $\hat{K}_1$ is a estimated parameter which satisfies the following adaptive law

$$\hat{K}_1 = |\hat{s}|, \hat{K}_1(0) = \hat{K}_{10} \quad (8)$$

where $\hat{K}_1(0)$ is the bounded positive initial condition of $\hat{K}$. When *K* is a constant value, a small change in the system parameters such as μ, will effect on the tracking errors. Using an adaptive gain, not only decrease sensitivity to the parameters but also, synchronization the master and the slave systems can finally realized for any initial value of $\hat{K}_1$

III. System Description

The Lorenz system is known to be a simplified model of several physical systems. At the origin, it was derived from a model of the earth's atmospheric convection flow heated from below and cooled from above [5]. Chen system is a typical chaos anti-control model, which has a more complicated topological structure than Lorenz attractor [6]. Lu system is a typical transition system, which connects the Lorenz and Chen attractors and represents the transition from one to the other [8]. These nonlinear differential equations will be described in the next section. Table I summaries the design parameters are used in this paper.

TABLE I: The Design Parameters

Parameters	Symbol
Sliding Surface	k_0, k_1, σ
Controller	$K_0, \hat{K}_1(0), \mu$
High Gain Observer	$\varepsilon, \alpha_1, \alpha_2$

IV. Synchronization Between Two Different Chaotic Systems

A. Chaos Synchronization Between Lorenz and Lu Systems

We assume that Lorenz system drives the Lu system. The drive system (Lorenz) is defined as follows

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1), a = 10 \\ \dot{y}_1 = cx_1 - x_1z_1 - y_1, c = 28 \\ \dot{z}_1 = x_1y_1 - bz_1, b = 8/3 \\ y_m = x_1, \end{cases} \quad (9)$$

And the equations of response system (Lu) is

$$\begin{cases} \dot{x}_2 = \rho(y_2 - x_2), \rho = 36 \\ \dot{y}_2 = -x_2z_2 + vy_2 + u, v = 20 \\ \dot{z}_2 = x_2y_2 - \mu z_2, \mu = 3 \\ y_s = x_2, \end{cases} \quad (10)$$

where y_m and y_s are the outputs of master and slave systems, respectively. Thus the relative degree is 2 (i.e. ρ= 2) and it is necessary to use the high gain observer (HGO)

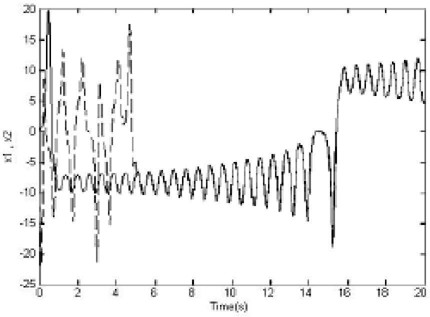
$$\begin{cases} \dot{\hat{e}}_1 = \hat{e}_2 + \alpha_1(e_1 - \hat{e}_1)/\varepsilon, \\ \dot{\hat{e}}_2 = \alpha_2(e_1 - \hat{e}_1)/\varepsilon^2, \end{cases} \quad (11)$$

$\alpha_1 = 15, \alpha_2 = 50, \hat{e}_1 = \hat{e}_2 = 0, \varepsilon = 0.01,$ is chosen. Now the sliding surface and controller is defined as follows

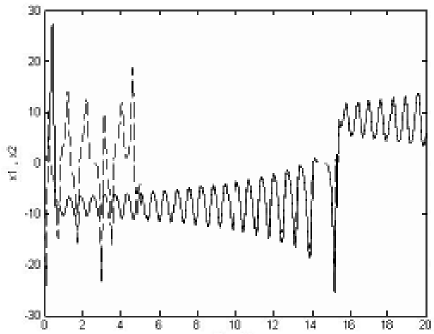
$$\begin{cases} \hat{s} = k_0\sigma + k_1e_1 + \hat{e}_2; e_1 = y_s - y_m, \\ \dot{\sigma} = -k_0\sigma + \mu sat(\hat{s}/\mu), \\ u = -(K_0 + \hat{K}_1)sat(\hat{s}/\mu), \\ k_0 = \hat{K}_1 = |\hat{s}|, \hat{K}_1(0) = K_{10}, \end{cases} \quad (12)$$

The initial values of Lorenz and Lu systems are $x_1(0) = 0.5, y_1(0) = -0.1, z_1(0) = -1$ and $x_2(0) = -10$

$y_2(0) = -11, z_2(0) = 5$ respectively. Constant $k_1(0) = 0.5$ is chosen such that the mentioned polynomial is Hurwitz. Parameters $\hat{K}_{10} = 10, K_0 = 40, \mu = 2, \sigma = 0$ were chosen. In Fig. 1, it shows that the slave and master systems can be reach synchronization with control operation. The controller is acted at *t*=5*s*.



(a)



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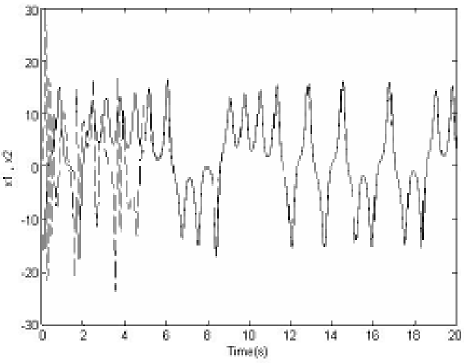


(c)

Fig. 1. Synchronization of Lorenz system (-) and Lu system (--): (a) shows the time response of signals x_1, x_2, (b) signals, y_1, y_2 (c) signals z_1, z_2

B. Chaos Synchronization Between Lorenz and Chen Systems

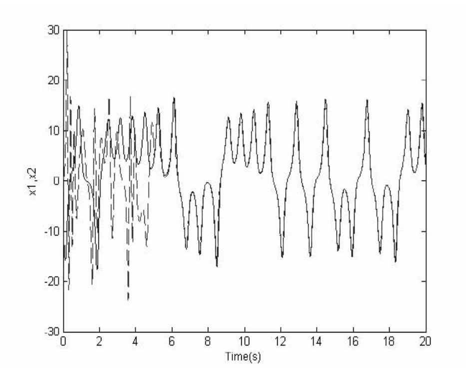
Our goal is to make synchronization between Lorenz and Chen system by using continuous sliding mode control. We assume that Lorenz system drives the Chen system. So the drive systems are described as follows

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1), a = 10 \\ \dot{y}_1 = cx_1 - x_1 z_1 - y_{1,} c = 28 \\ \dot{z}_1 = x_1 y_1 - bz_1, b = 8/3 \\ y_m = x_1, \end{cases} \tag{13}$$

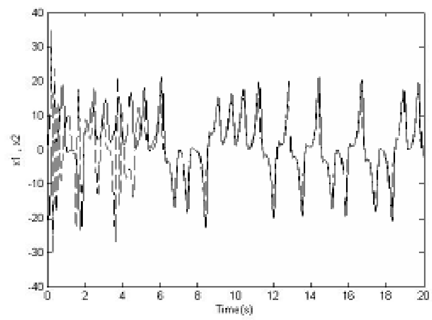
and the response system (Chen) is defined as follows

$$\begin{cases} \dot{x}_2 = \alpha(y_2 - x_2), \alpha = 35 \\ \dot{y}_2 = (\gamma - \alpha)x_2 - x_2 z_2 + \gamma y_2 + u, \gamma = 28 \\ \dot{z}_2 = x_2 y_2 - \beta z_2, \beta = 3 \\ y_s = x_2, \end{cases} \tag{14}$$

The equations show that the relative degree is 2 (i.e. ρ=2). Thus the high gain observer, the sliding surface and controller is defined as the last section. The initial values of Chen and Lorenz systems are $x_1(0) = -10, y_1(0) = -15, z_1(0) = 15$ and $x_2(0) = 0.2 y_2(0) = -0.5, z_2(0) = 1$. Parameters $\hat{K}_{10} = 10, K_0 = 50, \mu = 2, \sigma(0) = 0, k_1 = 0.5, k_0 = 1$ were chosen. The diagram of the Chen system controlled to be Lorenz system is shown in Fig. 2. The controller at t=5 s is applied.



(a)



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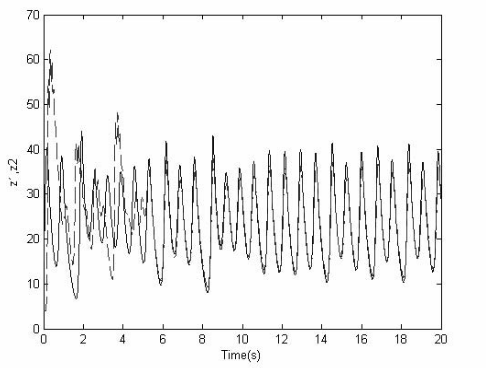
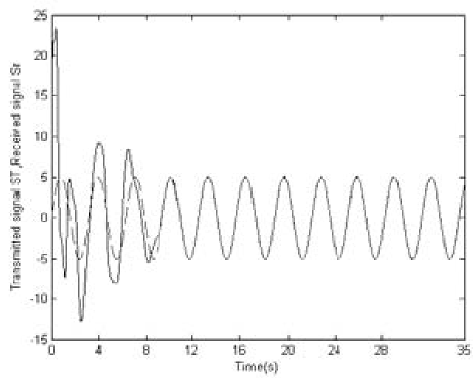


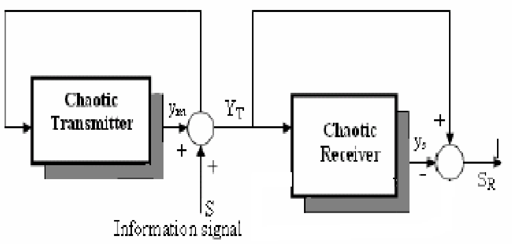
(c)

Fig. 2. Synchronization of Lorenz system (-) and Chen system (--): (a) shows the time response of signals x_1, x_2, (b) signals y_1, y_2 (c) signals z_1, z_2.

However, two chaotic systems are different, but simulations show that the slave system and the master system can reach synchronization in about 1 sec with control operation. After the synchronization of transmitter (drive) and receiver (response), one would like to know, if a message signal can be covered in spite of model differences between transmitter and receiver. Here we use the demodulation scheme reported in [10]. The transmitted signal is a sum of the information and the output of the chaotic transmitter, that is, $y_T = y_m + s(t)$ where S(t) is the information signal that masked by the system's output. By the proposed synchronization technique, a chaotic receiver is then derived to recover the information signal at the receiver end of communication. Fig. 3-b shows the typical frame of the communication. The information signal was chosen to be a periodic function S(t) =5sin2t. The frequency was chosen such that the dynamics behavior of the drive system remains chaotic. Using above system is detected the periodic signal. The controller is tuned at t=10 s. The message signals are decoded with acceptable accuracy. Information signal S(t) and recovered signal $S_R(t)$ is depicted in Fig.3-a.



(a)



(b)

Fig. 3. (a) Information signal S_T (--) and recovered signal S_R (-), (b) The typical frame of communication transmission

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Zahra Rabiei received her BSc degree in Electrical Engineering, Electronic branch from K.N.T University of Technology, Tehran, Iran in 200 , MSc degree in Control Systems Engineering from Ferdowsi University, Mashhad, Iran in 2005. She is currently a faculty member of Islamic Azad University of Aliabad Katoul, Golestan province, Iran. Her research interests are Nonlinear Control, Biomedical Control application and Robotics.

G.Reza Bidari received a BSc degree in Electrical Engineering, Electronic branch from Amirkabir University, Tehran, Iran in 1995 and MSc degree in Control Systems Engineering from Tarbiat Modares University, Tehran, Iran in 1999. He is currently a PhD student in Control Systems Engineering in Ferdowsi University of Mashhad, Iran. His interests include Nonlinear Control, Hybrid systems, Reinforcement Learning and Machine Learning.

Naser Pariz received his BSc degree in Electrical Engineering, from Ferdowsi University, Mashhad, Iran. He also received MSc degree and PhD from Ferdowsi university. Dr Naser Pariz is Associate Professor at department of Electrical Engineering of Ferdowsi University, Mashhad, Iran.
His research interests include Nonlinear Control, Hybrid Systems, Robust Detection and Control Theory.