Comparative Analysis of Matched Filter and Wavelets for Radar Noise Reduction

Md Saiful Islam and Uipil Chong

***Abstract*—This paper compares Matched filter and Wavelet transform applied to both Continuous Wave Radar and Pulse Radar signals to reduce the noise. For removing noise and extracting signal, Wavelet analysis is one of the most important methods. The denoising application of the wavelets has been used in spectrum cleaning of the atmospheric radar signals. Matched filter has strong anti-noise ability; it can also achieve accurate pulse compression in a very noisy environment. This paper analyzes the algorithms of Matched filter and Wavelets that are used in radar signal processing to reduce the noise. The simulation results indicate that Matched filter has strong anti-noise ability for Pulse Radar and Wavelet for Continuous Wave Radar.**

***Index Terms*—Denoising, matched filter, radar noise, SNR, wavelets.**

I. Introduction

Weak signal detection is a basic and important problems in radar systems. Traditionally, signals (encompassing desired signals as well as interfering signals) can be classified as deterministic signals, which waveforms defined precisely for all instants of time and stochastic processes, which is defined by an underlying probability distribution [1]. These two broadly defined classes overlook another important class of signals, known as chaotic signals which have very irregular waveform; but are generated by a deterministic mechanism [2]. A chaotic signal share attributes with both deterministic signals and stochastic processes [1].

Chaos is the very complicated behavior of a low-order dynamical system, because it is both nonlinear and deterministic [1]. It demonstrates a strong notion, permitting the use of a simple deterministic system to illustrate highly irregular fluctuations exhibited by physical phenomena encountered in nature. Recently, some engineering applications of chaos have been reported in literature [3]-[6]. These can be grouped under two broadly defined categories [3], [4]. One group is synthesis of chaotic signals which includes signal masking and spread-spectrum communications. Another is analysis of chaotic signals. It exploits the fact that some physical phenomena allow the use of a chaotic model.

Wavelet analysis [5] and Matched filter [6] are two of the most significant tools in the field of signal processing in the last few decades. We analyze both techniques and show how such an individual can improve the quality of radar-received signals in a noisy environment for both types of radar i.e. Continuous Wave Radar (radars continuously transmit a high-frequency signal and the reflected energy is also received and processed continuously) and Pulsed Radar (transmits high power, high-frequency pulses toward the target and it waits for the echo of the transmitted signal before it transmits a new pulse). The problem addressed here concerns the de-noising of the radar received signal immersed in noise. Several simulations have been performed to verify the algorithm for both types of radar. All of the simulations give the same time delay, and even the received signal is attenuated more than 90%. The simulation results indicate that algorithm is effective and robust even when the receiver receives a very weak signal.

The remainder of the paper is organized as follows:- In Section II, we briefly describe the literature of Matched filter and chracterisctics of Matched filter related to radar noise reduction. Section III presents wavelets denoising technique, while the simulations and data analysis are described in Section IV. Finally, we make some conclusions about our comparision related to noise reduction.

II. Noise Reduction by Matched Filter

A. Matched Filter

Matched filter is not a specific type of filter, but a theoretical frame work. It is an ideal filter that processes a received signal to minimize the effect of noise. Therefore, it optimizes the signal to noise ratio (SNR) of the filtered signal [7].

The radar received signal $r(t)$ contains two components $s_i(t)$ and $n_i(t)$ that represent the certain signal (e.g. targets) and noise respectively i.e. $r(t)=s_i(t)+n_i(t)$. The matched filter $h(t)$ yielding the output $y(t)=s_o(t)+n_o(t)$ is to generate the maximal ratio of $s_o(T)$ and $n_o(T)$ in the sampling values at time T. Where $s_o(t)$, $no(t)$ are the outputs of $s_i(t)$ and $n_i(t)$ respectively after the matched filter shown in Fig. 1.

In Fig. 1, $s_i(t)$ represents the certain signal(e.g. target signal) we have to detect, and $n_i(t)$ represents the additive white Gaussian noise in the system. $s_o(t)$ *and* $n_o(t)$ represent the output of the matched filter by $s_i(t)$ and $n_i(t)$. Matched filter maximized the SNR, the ratio of the power of $s_o(t)$ and power of $n_o(t)$ according to the Schwartz Inequality.

Here, we suppose that the noise $n_i(t)$ additive white Gaussian noise, whose power spectrum is $N/2$, and the spectrum of the target's signal $s_i(t)$ is [7].

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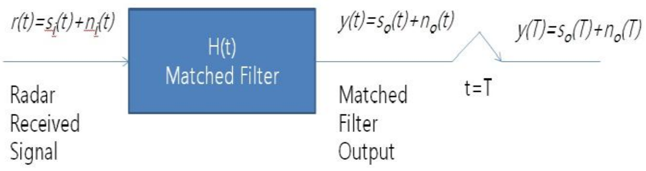

Fig. 1. Matched filter system.

$$F_{s_i}(\omega)=\int_{-\infty}^{+\infty} s_i(t)e^{-j\omega t}dt \qquad (1)$$

The above equation is the Fourier Transform of $s_i(t)$. The output of the matched filter $y(t)$ also contains two components representing the target's signal and noise respectively, $y(t)=s_o(t)+n_o(t)$, where

$$s_o(t)=\frac{1}{2\Pi}\int_{-\infty}^{+\infty}\left[H(\omega)F_{s_i}(\omega)\right]e^{j\omega t}dt \qquad (2)$$

The average power of the noise equals the value of auto -correlation function, which is

$$E[n^2{}_o(t)]=\frac{1}{2\Pi}\frac{N}{2}\int_{-\infty}^{+\infty}\left|H(\omega)\right|^2dt \qquad (3)$$

Now, according to the definition of the signal-noise ratio (SNR) at the moment T is

$$\text{SNR}=\frac{\left|s_o(T)\right|^2}{E(n^2{}_o)}$$

$$=\frac{\left|\frac{1}{2\Pi}\int_{-\infty}^{+\infty}\left[H(\omega)F_{target}(\omega)\right]e^{j\omega t}d\omega\right|^2}{\frac{1}{2\Pi}\frac{N}{2}\int_{-\infty}^{+\infty}\left|H(\omega)\right|^2d\omega} \qquad (4)$$

Using the Schwartz Equality, we get

$$\left|\frac{1}{2\Pi}\int_{-\infty}^{+\infty}\left[H(\omega)F_{target}(\omega)\right]e^{j\omega t}dt\right|^2$$

$$\leq\frac{1}{4\Pi^2}\int_{-\infty}^{+\infty}\left|H(\omega)\right|^2d\omega\int_{-\infty}^{+\infty}\left|F_{target}(\omega)\right|^2d\omega$$

Hence,

$$SNR\leq\frac{\frac{1}{4\Pi^2}\int_{-\infty}^{+\infty}\left|H(\omega)\right|^2d\omega\int_{-\infty}^{+\infty}\left|F_{target}(\omega)\right|^2d\omega}{\frac{1}{2\Pi}\frac{N}{2}\int_{-\infty}^{+\infty}\left|H(\omega)\right|^2d\omega}$$

$$=\frac{\frac{1}{2\Pi}\int_{-\infty}^{+\infty}\left|F_{target}(\omega)\right|^2d\omega}{\frac{N}{2}} \qquad (5)$$

The numerator of the above equation denotes power of the signal according to the Parseval's Theorem. From equation (5), the Matched filter maximizes the SNR of the filtered signal and has an impulse response that is a reverse time-shifted version of the input signal. So to obtain the maximum SNR, we need the time delay, D. With use of this time delay, D, we obtain the output of the cross correlation between transmitted signals and received signals.

B. Cross-Correlation to Find Time Delay D

Correlation is the process to determine degree of 'fit' between two waveforms and to determine the time at which the maximum correlation coefficient or "best fit" occurs [8]-[10]. For the radar system, if we correlate between the transmitted signal and the received signal, then we get the time difference between the transmitted and received signals. We consider the transmitted signal to be $x(n)$, and then the returned signal $r(n)$ may be modeled as:

$$r(n)=\alpha x(n-D)+w(n) \qquad (6)$$

where $w(n)$ is assumed to be the additive noise during the transmission, α is the attenuation factor, D is the delay which is the time taken for the signal to travel from the transmitter to the target and back to the receiver.

The cross-correlation between the transmitted signal, $x(n)$ and the received signal, $r(n)$ is [8]

$$C_{xr}(l)=\alpha C_{xx}(l-D) \qquad (7)$$

From equation (7), the maximum value of the cross-correlation will occur at $l=D$, which is our interest in cross-correlation from which we can get the time delay, D. For the multiple targets, we get the multiple number of D from equation (7). For example, if there are n targets then we can get n number of delay such as D_1, D_2, D_3, ……,. D_n.

C. Matched Filter for Radar

For Pulsed radars, consider pulse width τ_ρ, τ_κ is the time that a target is illuminated by the radars. Thus, we can write $r(t)$ as

$$r(t)=Vrect\left[\frac{t-\tau_\rho}{\tau_\kappa}\right] \qquad (8)$$

from equation (5), and for equation (8), we can write the pulse radar as [11]

$$(SNR)_{pulse}=\frac{P_tG^2\lambda^2\sigma\tau_\rho}{(4\pi)^3KT_oBFLR^4} \qquad (9)$$

where Pt is the peak transmitted power of radar, G is the antenna gain, σ is the Radar Cross Section (RCS), R is the range which electromagnetic wave transmits, λ is the wavelength, K=1.38×10-23J/K is Boltzmann's constant，T_0=290K is the operating temperature of antenna, F is the noise figure of receiver，L denotes as radar losses.

For Continuous Wave Radars, radar equation can be written as [11].

$$(SNR)_{cw} = \frac{P_{CW} T_{DWELL} G^2 \lambda^2 \sigma}{(4\pi)^3 K T_o B F L R^4} \qquad (10)$$

where P_{CW} is the Continuous Wave average transmitted power， T_{DWELL} is the dwell interval.

III. Wavelet De-Noising

For removing noise and extracting signal from any data, Wavelet analysis is one of the most important methods. The de-noising application of the wavelets has been used in spectrum cleaning of the atmospheric signals. There are different types of wavelets available like Morlet, Coiflets, Mexican Hat, Symlets, and Biorthogonal Haar, which have their own specifications such as filter coefficients and, reconstruction filter coefficients. In this paper, to eliminate noise embedded in the radar signal "sym8," wavelets have been used. The goal of this study is to denoise the radar signal. One often encounters the term ‘d*e-noising*’ in recent wavelet literature, describe in an informal way with various schemes that attempt to reject noise by damping or thresholding in the wavelet domain [12], [13]. The threshold of wavelet coefficient has near optimal noise reduction for different kinds of signals. Wavelets have a lot of advantages over fast Fourier transform. Fourier analysis has a major drawback, which is that time information is lost; when transforming to the frequency domain. Thus, it is impossible to tell when a particular event took place under Fourier analysis. Wavelet analysis is capable of revealing aspects of data that other signal analysis technique; aspects like trends, breakdown points, discontinuities in higher derivatives, and self-similarity are unable to reveal. Wavelet analysis can often denoise a signal without appreciable degradation. The Wavelet transform performs a correlation analysis. Therefore the output is expected to be maximal when the input signal most resembles the mother wavelet.

IV. Simulation and Data Analysis

The denoising of the received radar signal is simulated in the presence of white Gaussian noise. The effect of signal parameter changes on the algorithm has been investigated. These parameters include the SNR of the signal. The SNR is defined as the ratio of the signal power to the noise power in the entire period. The following parameters are assumed: sampling frequency=10 GHz, pulse duration=8 ns, pulse repetition frequency=.24 GHz for the pulse radar (Fig. 3) and transmitted frequency= 10 GHz for continuous wave radar (Fig. 2). The receiver receives the return from the targets in the present of AWGN.

We recover our transmitted signal from the received signal using the matched filter for both Pulse Radar and Continuous wave Radar. For the Pulse Radar, the recovery signal is almost similar to transmitted signal shown in Fig. 2. (c). But in the case of Continuous Wave Radar, it is almost impossible to understand the shape of the transmitted signal from the recovery signal (Fig. 3). Fig. 4 and Fig. 5 are similar to Fig. 2 and fig. 3 respectively except the received signals are attenuated more than 90%. the Fig. 4 indicates that the Matched filter has a strong advantage in anti-noise ability for Pulse Radar. Unfortunately, the Matched filter for Continuous Wave Radar is not superior to recover transmitted signal (Fig. 5 (c)). The reason for this is that the transmitted average power of the Continuous Wave Radar is lower, and the thermal noise relevant to the operating bandwidth and the radar‘s structure design, so the signal detection threshold of Continuous Wave Radar is lower, and demand higher receiver sensitivity (Table I). This is shown in Fig. 6 using equation (9) and equation (10).

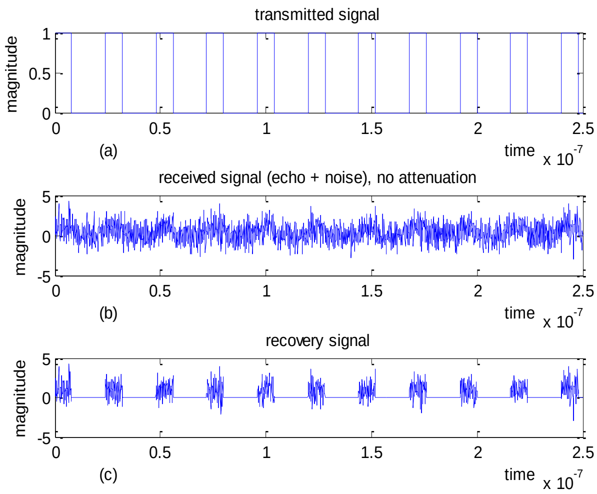

Fig. 2. (a) Pulse radar transmitted signal, (b) Received signal (echo + noise), no attenuation (c) Recovery signal from received signal by matched filter.

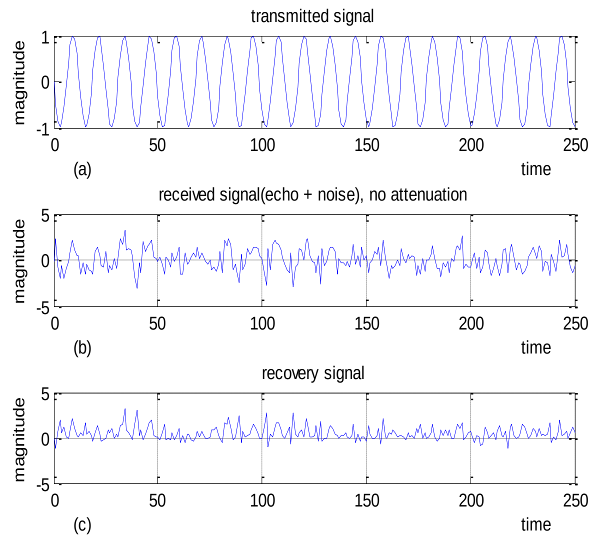

Fig. 3. (a) Continuous Wave Radar transmitted signal, (b) Received signal (echo + noise), no attenuation (c) Recovery signal from received signal by matched filter.

We also apply the wavelet de-noising technique to remove the noise from received signal. This technique overcomes the problem of Continuous Wave Radar. Fig. 7 (c) shows the improvement in noise reduction by Wavelet for Continuous Wave Radar. By Comparing this with Fig. 3 (c), we can say that more, noise is reduced in Fig. 7 (c). The reason for this is that the SNR is increased (Table I) when we use the wavelet denoising technique for Continuous Wave Radar. This is shown in Fig. 8.

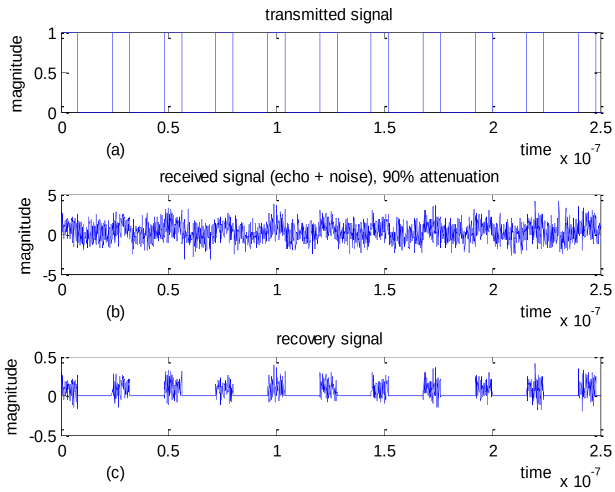

Fig. 4. (a) Pulse Radar transmitted signal, (b) Received signal (echo + noise), 90% attenuation (c) Recovery signal from received signal by matched filter.

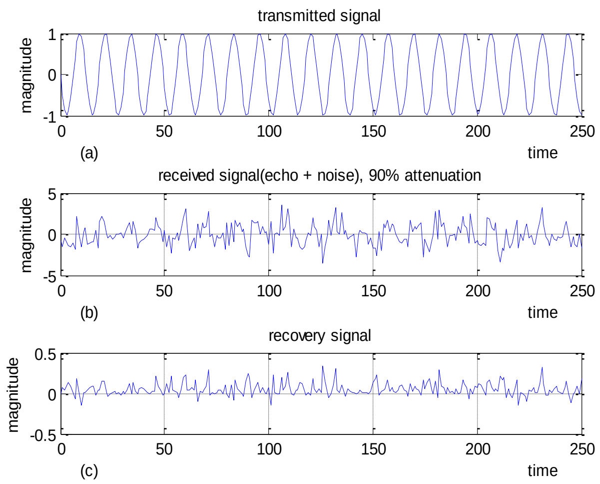

Fig. 5. (a) Continuous Wave Radar transmitted signal, (b) Received signal (echo + noise), no attenuation (c) Recovery signal from received signal by matched filter.

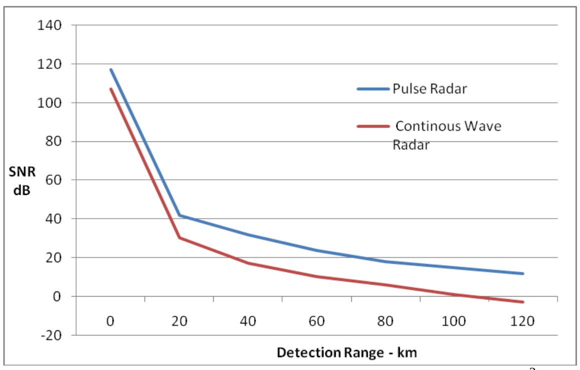

Fig. 6. Plots of SNR versus detection range *RCS*=1000 m^2.

TABLE I: SNR OF PULSE RADAR AND CONTINUOUS WAVE RADAR

Range in Km	SNR-dB		
	Pulsed Radar(Matched filter)	CW Radar (Matched filter)	CW Radar (Wavelet)
0	117	107	114
20	42	30	40
40	32	17	22
60	24	10	14
80	18	6	9
100	15	1	4
120	12	-3	1

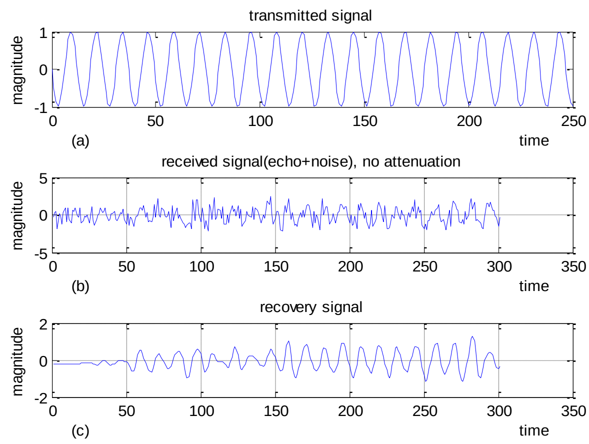

Fig. 7. (a) Continuous Wave Radar transmitted signal, (b) Received signal (echo + noise), no attenuation (c) Recovery signal from received signal by Wavelet.

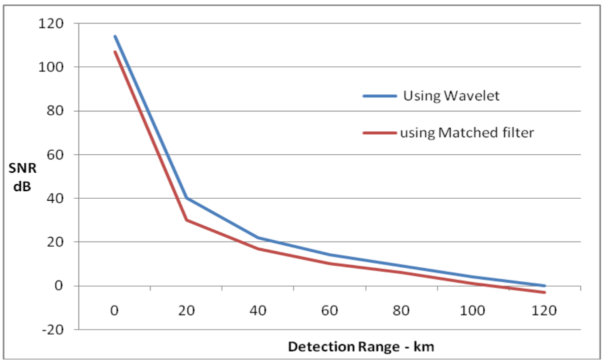

Fig. 8. Plots of SNR versus detection range *RCS*=1000 m^2 for continuous wave radar.

V. CONCLUSIONS

In this paper, the noise reduction of Pulse Radar and Continuous Wave Radar have been studied by matched filter and Wavelet. Matched filter has a distinct advantage in anti-noise ability. A significant reduction in noise is achieved for Pulse Radar by Matched filter, but employing Matched filter for Continuous wave increases the difficulty of detection. The wavelet de-noising technique even further reduced more noise for Continuous Wave Radar in Comparison to the Matched filter.

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