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An Iterative Parameters Estimation Method for Hyperbolic Frequency Modulated Signals with Colored Noise

Shuai Yao and Shiliang Fang

***Abstract*—This paper presents an iterative method for estimating the starting frequency and period slope of hyperbolic frequency modulated (HFM) signals with underwater acoustic platform colored noise. The method involves, firstly, whitening of the colored noise and, secondly, instantaneous frequency (IF) estimation of HFM signals based on the peak of short-time Fourier transform (STFT) and taking reciprocal of the estimated IF to get the zero crossing interval (ZCI). Parameters estimation of HFM signals is then achieved by using iteratively reweighted least squares (IRLS) linear fitting method to fit the ZCI. The main contributions of our work are three-fold: (1) A simple and effective method is proposed to whiten the underwater acoustic platform colored noise; (2) A complex nonlinear problem is transformed into a linear one by the proposed parameters estimation method; (3) The proposed method is applicable for the practical engineering applications.**

***Index Terms*—Colored noise, hyperbolic frequency modulated signals, short-time fourier transforms, iteratively reweighted least squares.**

I. Introduction

With the inherent Doppler-invariant property, HFM signals have been widely used in sonar systems for detecting moving target, especially for detecting small target moving at high speed, such as torpedo [1], [2]. The starting frequency and period slope are two basic parameters characterizing the HFM signals, so developing a fast and robust estimated method for these two parameters is crucial for efficiently confronting active sonar system using HFM signals.

The parameters estimation for HFM signals has been investigated roughly at present. In [3], the maximum likelihood estimation (MLE) was proposed. In [4], the nonlinear least-square (NLS) matching approach was proposed. The technique based on the peak of time-frequency distributions (TFD) was proposed in [5]. Both the MLE and NLS method are nonlinear and do not exist in the closed form for HFM parameter estimation, so they are very computationally intensive and hard to realize in engineering applications. In addition, all these present methods did not consider the colored noise, such as the underwater acoustic platform noise.

The main goal of this paper is to develop a fast and robust method for parameters estimation of HFM signals received by the underwater acoustic platform. There exist two difficulties: firstly, the received signal is noised by the acoustic platform noise which makes it hard to detect the HFM signal; secondly, the received signal is distorted by the ocean ambient noise and multipath propagation through underwater acoustic channel. The proposed estimation method combines the technique based on the peak of STFT (representing a very fast and efficient approach) and IRLS linear fitting (representing a robust method). The proposed local signal to noise ratio (SNR) of the observed power spectrum density (PSD) for each short time window of STFT is made to suppress the underwater acoustic platform colored noise before estimating the IF. The IRLS linear fitting method is used to overcome the impact of outliers existed in the estimated IF caused by the ocean ambient noise and multipath propagation through underwater acoustic channel.

The remaining of this paper is organized as follows. The parameters estimation flow chart will be briefly introduced in Section II. The whitening method for the platform colored noise will be given in Section III. In Section IV, we will derive and describe the proposed parameters estimation method. The simulation results will be displayed and discussed in Section V, followed by our conclusion in Section VI.

II. Parameters Estimation Flow Charts

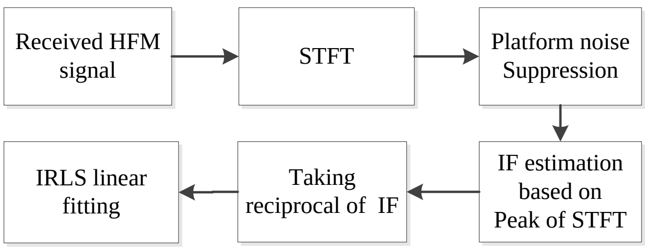

Fig. 1. Flow charts of HFM signal parameters estimation.

As shown in Fig. 1, after suppression of the platform noise for the STFT of the received HFM signal, parameters estimation for HFM signals is carried out by the proposed method combining STFT and IRLS linear fitting. The IF is estimated based on the peak of STFT which is less sensitive to the noise influence than the higher-order techniques [6], such as Wigner Ville Distribution (WVD). The STFT can be achieved by fast Fourier transform (FFT) which represents a very efficient and commonly applied approach. Then the ZCI is obtained by taking reciprocal of the estimated IF. At last, the IRLS linear fitting method is used to achieve the starting frequency and period slope estimation.

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III. Platform Nosie Suppression

The signal received by the hydrophones of passive sonar installed in the underwater acoustic platform is not only distorted by the ocean ambient noise and multipath propagation through underwater acoustic channel but also by the underwater acoustic platform noise, especially for the low frequency signals as the underwater acoustic platform noise concentrated in the low frequency. The underwater acoustic platform noise is colored. The slope in the observed PSD can reach up to 6 dB per octave [7]. To pick up the actual line spectrum from the magnitude spectrum of HFM signal, the underwater acoustic platform noise needs to be well suppressed.

The underwater acoustic platform noise can be described by the Esc noise model [8]. The PSD function of the Esc noise model can be expressed as

$$P_c(f)=\frac{\sigma^2}{2\pi}\left[\frac{f_m+\lambda(f+f_p)}{f+(f+f_p)^2}+\frac{f_m-\lambda(f-f_p)}{f+(f-f_p)^2}\right] \tag{1}$$

It can be seen from (1) that the PSD function of the Esc noise model depends on three parameters f_p, f_m and λ. The position of the spectral peak of the PSD function is decided by parameter f_p while the shape of the spectral peak depends on f_m. λ is used to control the power rate between the low and high frequency. f_p, f_m and λ need to satisfy the condition that $f_{\mathrm{m}}<\left(\sqrt{3}/3\right)f_{\mathrm{p}}$ and $|\lambda|\le\sqrt{3}/3$.

The basic PSD shape is shown in Fig. 2(a). It can be seen from Fig. 2(a) that the PSD of the Esc noise firstly increases and then decreases with the frequency increases. It is one of the typical colored noises.

The PSD of the received (observed) signal with the underwater acoustic platform noise and Gauss white noise is defined as

$$P(f)=S(f)+P_c(f)+N(f) \tag{2}$$

where $S(f)$ is the PSD of the transmitted signal of the active sonar and $N(f)$ is the PSD of the Gauss white noise.

The PSD of a received (observed) single-frequency signal is shown as the solid line in Fig. 2(b). It can be seen from Fig. 2(b) that the signal line spectrum is far lower than the peak of $P(f)$, so it is hard to pick up the signal spectral line from the PSD of the received signal. To suppress the influence of the underwater acoustic platform noise, it is needed to whiten the PSD of the underwater acoustic platform noise while keep the line spectral characteristic of the transmitted signal.

A method called as local SNR method in this paper is proposed to achieve whitening the PSD of the underwater acoustic platform noise. The local SNR for the PSD of the received signal is defined as

$$P_{snr}(f)=\frac{P(f)}{\int_{f-f_b/2}^{f+f_b/2}P(f')df'-P(f)} \tag{3}$$

where f_b is the frequency bandwidth to calculate the local SNR of the observed PSD.

It can be seen from Fig. 2.(b) that the underwater acoustic platform noise is well suppressed while the signal spectral line of the PSD is kept well. The peak of the $P_{snr}(f)$ coincides with the signal line spectrum. It is very easy to pick up the signal spectral line based on the peak of STFT from the local SNR of the PSD of the received signal. In addition, (3) is easy to carry out for the PSD of the received signal and it needs a very small amount of computation amount.

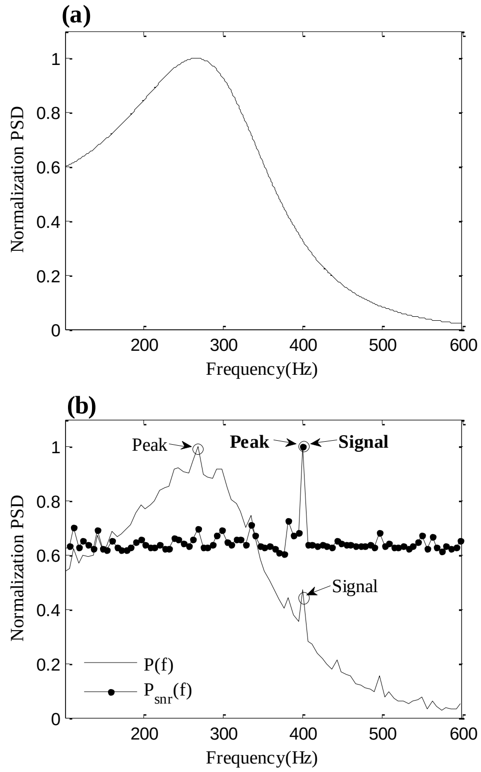

Fig. 2. Normalization PSD (a) Normalization $P_c(f)$ (b) Normalization $P(f)$ and $P_{snr}(f)$.

IV. Parameters Estimation Method

A. ZCI Estimation

The complex form of HFM signal is defined by

$$\begin{aligned} s(t) &= A\exp[\varphi(t)] \\ &= A\exp\left\{ j[-\frac{2\pi}{k_0}\ln(-k_0 t+1/f_1)]\right\} \end{aligned} \tag{4}$$

where $0<t<\tau$, $\varphi(t)$ is the instantaneous phase, A is the magnitude, f_1 is the starting frequency, τ is the pulse duration and k_0 is a constant factor defined as period slope, given by $k_0=(f_2-f_1)/(\tau f_1 f_2)$, where f_2 represents the end frequency.

The IF of the HFM signal is defined by

$$f(t)=\frac{1}{2\pi}\frac{d\varphi(t)}{dt}=\frac{1}{-k_0 t+1/f_1},\quad 0<t<\tau \tag{5}$$

The ZCI of the HFM signal can be given by

$$\mathrm{g}(t)=\frac{1}{f(t)}=-k_0 t+1/f_1,\quad 0<\mathrm{t}<\tau \tag{6}$$

As shown by (6), the ZCI is a linear function of time. If we can estimate the IF of HFM signals and then take reciprocal of the estimated IF to get the ZCI, the parameters can be further estimated by linear fitting.

The IF estimation based on the peak of STFT is achieved by taking the frequency corresponding to the maximal line spectrum of the local SNR for the PSD of each short time window as the estimated IF for the HFM signal in the *kth* short time window. It is given by

$$\hat{f}_k=\frac{\arg\max_f\left[P_{snr}^k(f)\right]}{M}f_s,\quad k=1,2...K \tag{7}$$

where $P_{snr}^k(f)$ is the local SNR for the PSD of the *kth* short time window of the received signal, M is the short time window length of the STFT, $K=\mathrm{int}(N/L)$ is the total number of short time window, L is the moving step of the short time window. Then the ZCI can be estimated by taking reciprocal of the estimated IF,

$$\hat{g}_k=1/\hat{f}_k,\quad k=1,2...K \tag{8}$$

B. IRLS Linear Fitting

If there was no outlier, the conventional least squares (CLS) linear fitting is the optimal estimation. However, it is unavoidable that there exist outliers in the estimated ZCI which are caused by the ocean ambient noise and multipath propagation through underwater acoustic channel. When there are outliers, the CLS linear fitting will deteriorate dramatically. In order to reduce the impact of outliers in the estimated ZCI for the parameters estimation, we improved the IRLS linear fitting method proposed in [9] to adjust to the linear fitting of the ZCI of HFM signals. The improved IRLS linear fitting method is based on the CLS linear fitting and then iteratively reweighted least squares method is used to reduce the impact of outliers for the parameter estimation.

The proposed improved IRLS linear fitting algorithm is described formally as in the following:

Step 0: Compute the least squares linear fitting

$$(\hat{a}^0,\hat{b}^0)=\arg\min_{(a,b)}\sum_{k=1}^{I}[\hat{g}_k-(a+bk)]^2 \tag{9}$$

Step q: $q=1,2\ldots Q$

Compute the new weights w_k^q ,

$$\begin{aligned}d_k^q&=\left|\hat{g}_k-\hat{a}^{q\text{-}1}-\hat{b}^{q\text{-}1}k\right|\\ \lambda_k^q&=(d_k^q-d_{\min}^q)/(d_{\max}^q-d_{\min}^q)\\ w_k^q&=1/(\lambda_k^q+\delta)\end{aligned} \tag{10}$$

where k=1, 2, 3, … K, $d_{\min}^q$ is set to the minimum value of the $\{d_1^q,d_2^q,\ldots d_K^q\}$, $d_{\max}^q$ is set to maximum value of the $\{d_1^q,d_2^q,\ldots d_K^q\}$ and δ is the correction factor for the weights.

It is added to avoid the weights become infinite when λ_i^q is equal to 0.

Compute $\hat{a}^q$ and $\hat{b}^q$,

$$\hat{a}^q=\frac{\sum_{i=1}^{K}i^2w_i^q\sum_{j=1}^{K}w_j^q\hat{g}_j-\sum_{i=1}^{K}iw_i^q\sum_{j=1}^{K}jw_j^q\hat{g}_j}{\sum_{i=1}^{K}\sum_{j=1}^{K}w_j^qw_i^q(j^2-ij)} \tag{11}$$

$$\hat{b}^q=\frac{\sum_{i=1}^{K}w_i^q\sum_{j=1}^{K}jw_j^q\hat{g}_j-\sum_{i=1}^{K}iw_i^q\sum_{j=1}^{K}w_j^q\hat{g}_j}{\sum_{i=1}^{K}\sum_{j=1}^{K}w_j^qw_i^q(j^2-ij)} \tag{12}$$

The iteration is carried on till satisfing the stopping rule

$$\hat{q}=\min_q\left\{q:|J_w^q-J_w^{q-1}|\le\varepsilon J_w^{q-1},\ \ q\le Q\right\} \tag{13}$$

where $\varepsilon>0$ and $Q\ge1$ are set in advance. Experiments have shown fast convergence of the proposed method.

At last, the estimated start frequency $\hat{f}_1$ and period slope $\hat{k}_0$ are given by

$$\hat{f}_1=1/\hat{a}^{\hat{q}}\qquad \hat{k}_0=\hat{b}^{\hat{q}}/\left(L/f_s\right) \tag{14}$$

From the above algorithm procedure, it can be seen that, in step 0 of the proposed method, the fit is obtained by least squares linear fitting. In the iteration step q ($q=1,2\ldots Q$), the weighted linear fitting is used and the outliers and normal observations can be automatically identified and different weights are given to them. As λ_i^q is defined as relative values of the difference between the estimated ZCI and the fitted values of the last iteration, the weights w_i^q do not depend on the absolute values of the observations. It makes the proposed method in this paper to be more robust than the method in [9] which depends on the absolute values of the observations.

V. Smiulations

The simulated received signal is generated according to the real discrete form

$$x(n)=A\cos\left[-\frac{2\pi}{k_0}\ln\left(\frac{-k_0}{f_s}n+1/f_1\right)\right]+w_{\mathrm{c}}(n)+w_{\mathrm{w}}(n) \tag{15}$$

where $n=0,1,2...N-1$, $N=\mathrm{int}(f_s\tau)$, f_s is the sampling frequency, $w_{\mathrm{c}}(n)$ is the simulated underwater acoustic platform colored noise produced according to the Esc noise model given by (1), $w_{\mathrm{w}}(n)$ is an independent and identically distributed sequence of Gaussian random variables with zero mean and variance σ^2 .

The HFM signal is of length $N=2048$ samples, a sampling frequency $f_s=3$ KHz, a starting frequency

$f_1 = 250$ Hz, an end frequency $f_2 = 450$ Hz and amplitude $A=1$, so the period slope k_0 is 2.6×10^{-3}. The three parameters of the simulated colored noise are respectively f_m=120 Hz, f_p=300 Hz and λ=0.5. We assess the performance of the algorithm by plotting the mean-absolute-relative error (MARE) and normalized mean square error (NMSE) versus different SNRs starting from -12 to 12dB in a step of 1 dB.

The MARE and NMSE are respectively defined as

$$MARE(\hat{\mu}) = \frac{1}{N_r}\sum_{r=1}^{N_r}\frac{\left|\hat{\mu}_r - u\right|}{\mu} \quad (16)$$

$$MSE(\hat{\mu}) = \frac{1}{N_r u^2}\sum_{r=1}^{N_r}(\hat{\mu}_r - u)^2 \quad (17)$$

where $\hat{\mu}_r$ denotes the estimated parameter of the *rth* Monte Carlo run, μ denotes the actual value of the parameter and N_r is the total number of Monte Carlo runs.

In this paper, N_r=1000 Monte Carlo simulations were run for each SNR. For comparison purposes, the method of [4] and [9] were also simulated. Fig. 3(a) and Fig. 3(b) shows the MAREs of the estimated parameters $\hat{f}_1$ and $\hat{k}_0$, while their NMSEs are presented in Fig. 4(a) and Fig. 4(b).

From both Fig. 3 and Fig. 4, it can be seen that, for $SNR < -5dB$, the MAREs and NMSEs of starting frequency $\hat{f}_1$ and period slope $\hat{k}_0$ estimated by the proposed algorithm decrease rapidly with the SNR increases for $SNR < -5dB$. However, they exhibit a nearly flat value for $SNR > -5dB$. It is caused by that the STFT gives biased estimates of the IF. It can be improved by adaptive STFT but a high increase in computational load is needed.

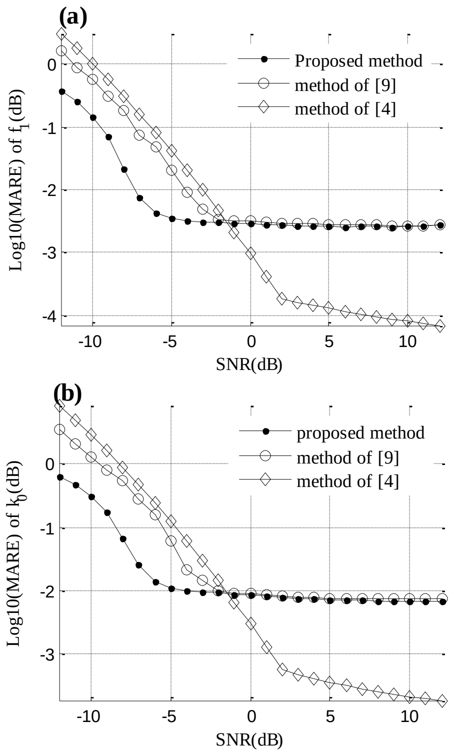

Fig. 3. MARE of $\hat{f}_1$ (a) and $\hat{k}_0$ (b) versus SNR.

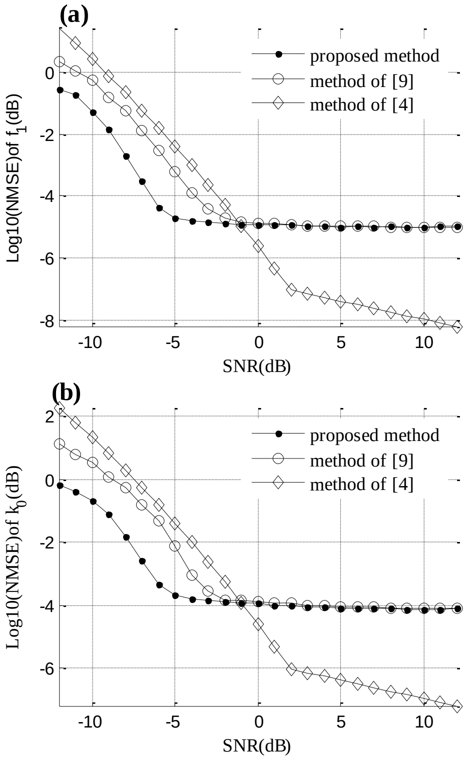

Fig. 4. NMSE of $\hat{f}_1$ (a) and $\hat{k}_0$ (b) versus SNR.

Fig. 3 and Fig. 4 also show that, for $SNR < -1dB$, the proposed method outperforms the presenting methods. Although the accuracy of the estimated parameters is not better than the method of [4] for $SNR > -1dB$, but the proposed method performs an evident decrease in calculation complexity. In addition, from Fig. 3(a) and Fig. 3(b), it can be seen that both the MAREs of the starting frequency $\hat{f}_1$ and period slope $\hat{k}_0$ estimated by the proposed algorithm are less than 0.01 for $SNR > -1dB$. It already meets most of the practical engineering applications. Therefore, the proposed method is applicable for estimating the parameters of the HFM signal with colored noise fast and robustly.

VI. Conclusion

An iterative parameters estimation method for HFM signals with underwater acoustic platform colored noise has been presented in this paper. Firstly, the underwater acoustic platform noise can be well suppressed while the signal line spectral is highlighted by the proposed local SNR method. The local SNR method can also be generalized to whiten other colored noises rather than just the underwater acoustic platform noise. Then the method combining STFT and IRLS linear fitting is used to estimate the parameters for the HFM signal based on the observed STFT processed by the local SNR method. The proposed estimation method requiring no parameter search has advantages of low complexity, small computational load and fast convergence and the accuracy of the estimated parameters meets most of the practical engineering applications. Therefore, the proposed method is applicable for estimating the parameters of the HFM signal with colored noise fast and robustly.

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