A Pre-Interleaver and Error Control Based Selective Mapping Method for PMEPR Reduction in MC-CDMA

Sajjad A. Memon, Zhe Chen, and Fuliang Yin

Abstract—The multi-carrier transmission signal in MC-CDMA has a high Peak-to-Mean Envelope Power Ratio (PMEPR) which results in nonlinear distortion and system performance degradation. To reduce PMEPR, a pre-interleaver and error control coding based selective mapping method is proposed in this paper. This method generates 2^n sequences of the original data sequence via adding n PMEPR control bits to the original data followed by an interleaver and error control code to make original data more random. The covolutional codes, cyclic codes and low density parity check (LDPC) codes are used as error control codes. The proposed method achieves significant PMEPR reduction and avoids the need of side information transmission. The simulation results reveal the validity of proposed method.

Index Terms—MC-CDMA, peak to average power ratio (PAPR), high power amplifier (HPA), selective mapping.

I. INTRODUCTION

Multi-Carrier Code division Multiple Access (MC-CDMA) system, also known as Orthogonal Frequency Division Multiplexing-Code Division Multiple Access (OFDM-CDMA) system, is a transmission technique proffering many alluring properties, such as high spectral efficiency and low receiver complexity, which makes it a promising candidate for the next-generation mobile radio systems [1]. In MC-CDMA, orthogonal codes are used to spread symbols of users and combine them in the frequency domain; this results in a relatively low symbol rate and non-selective fading in each subcarrier [2]. However, an inherent property of multi-carrier transmission schemes, including MC-CDMA, is high Peak-to-Mean Envelope Power Ratio (PMEPR) also referred as Peak to Average Power Ratio (PAPR) in literature [2]-[4]. A high PMEPR may result in a nonlinear distortion at high power amplifier (HPA) and increase the bit error rate (BER). To avoid the nonlinear distortion, the nonlinear amplifier need to operate close to linear region, which results in significant power efficiency penalty and makes transmitters very expensive [3], [4]. Therefore, the PMEPR reduction techniques for multi-carrier transmission schemes are very stringent.

A number of PMEPR reduction methods have been proposed in past decade years [5]. Among these methods, the distortionless methods are very attractive, since the information in transmitted signals is undistorted [6]. The Partial Transmit Sequences (PTS) method [7] and the Selective Mapping (SLM) method [8], [9] are the two typical distortionless ones. The key of PTS method is the optimal combination of phase-rotated signal sub-blocks to minimize the peak power of the transmitted signal, while in the SLM method, the frequency domain data is multiplied by a set of statistically independent sequences, and the corresponding time domain signal with the smallest PMEPR is selected and transmitted. Both methods provide the improved PMEPR performance, but need the side information to recover the original OFDM signal in the receiver. Although these methods were originally proposed for OFDM system, but can be implemented on MC-CDMA with minor modifications [10]-[13]. Moreover, the SLM method is more effective in reducing PMEPR than the PTS method at the same amount of side information, because in the PTS the phase must be rotated by clusters, whereas in the SLM method the phase is rotated only by one subcarrier. Thus, the probability of low PMEPR using the SLM is higher than the PTS [11], [14].

In [11]-[13], the SLM method was used to reduce the PMEPR for MC-CDMA. In [11] different sequences was examined as phase sequences and simulation results showed that the SLM method using random sequence as phase sequence is most effective in reducing PMEPR. In [12], the SLM method along with Selected Spread Code is used, where Walsh-Hadamard Transform (WHT) was employed as phase sequence and Walsh-Hadamard codes as spreading code. In code selection process, the transmitter selects the spreading code of each user from a set of Walsh-Hadamard codes, and the code that provide smallest PAPR after IFFT will be assigned to each user to minimize the output peak power in each symbol. In [13], a new pseudo random interferometry code sequence was used as the phase sequence for the SLM method to reduce the PMEPR in MC-CDMA.

In [8], [9], [11]-[13], the SLM method was used either with different phase sequences or with some spreading code selection algorithm, but all these need to send side information to the receiver regarding either which phase sequence or spreading code having the lowest PMEPR.

In this paper, an improved selective mapping method with an interleaver and error control code is proposed. The proposed method generates the 2^n data sequences of the original data sequence by adding n PMEPR control bits to the original data sequence followed by an interleaver and error control code. The use of interleaver and error control code makes each sequence more random, distant and maximize the PMEPR reduction. The covolutional codes, cyclic codes and low density parity check (LDPC) codes are used as error control codes in this paper. The propose method uses error control codes to increases the error correction capability at the receiver with the primary advantage of avoiding the need of side information to recover the original data at the receiver.
II. MC-CDMA TRANSMISSION SCHEME

MC-CDMA is a multi-carrier transmission scheme where the different users share the same bandwidth at the same time and separate the data by applying different specific user codes, i.e. the separation of the user signals is carried out in code domain. The principle of MC-CDMA is to allocate the chips of a spread data symbol in frequency direction over several parallel sub-channels and transmit a data symbol of a user simultaneously on several narrowband sub-channels.

The complex data symbol as the ratio of the peak power to the average power of the MC-CDMA signal, and mathematically it can be expressed as

\[ PMEPR = \frac{\max |x(n)|^2}{1/N \sum_{n=0}^{N-1} |x(n)|^2} \]  

where \( E(\cdot) \) is the expectation operator.

An MC-CDMA baseband signal is the sum of many data symbols modulated onto sub-channels. When the samples from all sub-carriers are added constructively, the peak power of the signal becomes \( N \) times the average power, thus results in high PMEPR. An high PMEPR signal requires a wide dynamic range in the high power amplifier at the transmitter. If the dynamic range in the high power amplifier is insufficient, the signal could be distorted from the resulting nonlinearity, which degrades the signal quality, and leads to out of band (OoB) radiations and hence interfere with adjacent frequency bands, which results in increase of bit error rate. To mitigate both nonlinear distortion and increase in bit error rate, the nonlinear amplifier needs to operate near the linear region with a large dynamic range. However, this results in significant power efficiency penalty and makes transmitters very expensive. Since power efficiency is very important in wireless communication, it is necessary to aim at a power efficient operation of the non-linear HPA with a low back-off values. Hence, a better solution for reducing PMEPR of the transmitted signal is with some manipulations of the multi-carrier signal itself. Since the user data is random in nature, it is necessary to evaluate the statistical characteristics of the PMEPR. The most classical approach for the analyses of PMEPR is to use Complementary Cumulative Distribution Function (CCDF), which is defined as the probability of the PMEPR exceeding a certain level \( z \) [5], [17], [18], i.e.

\[ P\{PMEPR > z\} = 1 - P\{PMEPR \leq z\} = 1 - \left( 1 - e^{-z} \right)^N \]  

IV. SELECTED MAPPING METHOD

Consider frequency domain data of MC-CDMA system with \( L \) sub-carriers is \( X_m(n=0, \ldots, L-1) \). In the conventional SLM method the phase sequences \( Q^{(u)} = [Q^{(u)}_0, Q^{(u)}_1, \ldots, Q^{(u)}_{L-1}] \) \((u=1, \ldots, U)\) are multiplied element-wise with \( X_m \) to generate \( U \) candidates \( X^{(u)}_m \) [8], i.e.,

\[ X^{(u)}_m = Q^{(u)} X_m, \quad m=0, \ldots, L-1 \]  

where \( Q^{(u)} = \exp(j \phi^{(u)}_m) \) \((m=0, \ldots, L-1)\), and \( \phi^{(u)}_m \) is randomly selected from \([0, \pi/2, \pi, 3\pi/2]\). To preserve the power, each element of the phase sequence has unit magnitude.

Finally, the time domain candidate \( X^{(u)}_t \) with the minimum PMEPR is elected for transmission with the side information about the phase sequence. At the receiver, the reverse rotation is performed to recover the data.

V. PRE-INTERLEAVER AND ERROR CONTROL SLM (PIE-SLM)

In this section, a new approach that uses SLM with Error
Control (EC) codes is proposed, which has the same principle as the SLM method for PMEPR diminution [8], but consists on robust EC ability, and eliminates the error propagation and the requirement of side information transmission.

Fig. 2(a) shows block diagram MC-CDMA transmitter using PIEC-SLM method. In PIEC-SLM method U candidates are generated by adding n PMEPR control bits to the spread data that results in U=2^n candidates which are statistically independent spread data sequences as shown in Fig. 2(a). Then, U candidates are interleaved, followed by error control coding, which makes U candidates more random from each other and maximizes PMEPR reduction. In this paper, convolutional codes, cyclic codes and low-density-parity-check (LPDC) codes are used as error control codes. Finally, a time domain candidate is selected with minimum PMEPR for transmission without the requirement of side information.

Fig. 2(b) shows block diagram of MC-CDMA receiver using the PIEC-SLM method. In Fig. 2(b), spread data sequence with minimum PMEPR value is received and passed through FFT, parallel to serial conversion, demodulation, error control decoding and de-interleaving. After de-interleaving, the n PMEPR control bits are detached from spread data sequence, followed by de-spreading using user specific code, which yields user data d(k) that was payload. In the propose, method the receiver do not require side information to recover the transmitted data.

VI. ERROR CONTROL CODES

A. Convolutional SLM Codes [19]

Convolutional codes are different from block codes in that the encoder contains memory and the o outputs at any given time unit depends on not only the k inputs at that time unit, but also the inputs of K previous time units. The output sequence U is equal to the convolution of the input sequence m and the encoder’s impulse response. Due to encoder memory effect, the input m and output U of convolutional codes are not in blocks but continuous data sequences. Fig. 3 illustrates the general form of convolutional encoder.

Parameters and notations of general convolutional encoder in Fig. 3 are the following:

- Input data sequence m=m_1, m_2, m_3, ...
- Input dimension k: data bits are shifted into the encoder k bits at a time.
- Output codeword sequence U=U_1, U_2, ..., U_o, where U_i=u_{i1}, u_{i2}, ..., u_{io} is the i-th codeword branch, here u_{ij} is the j-th bit of the i-th codeword branch.
- Output dimension o: output bits are produced o bits at a time simultaneously.
- Constraint length K: number of stages in the shift register
- Code rate R=k/o.

Convolutional codes have three flavors, corresponding to the three types of filters, that is, Systematic Nonrecursive (SN), Nonsystematic and Nonrecursive (NN) and Systematic Recursive (SR). SN has the property that one of the output bits is identical to the input bit. This encoder is thus called systematic and nonrecursive because of no feedback. NN also has no feedback, but it is not systematic. It makes use of two tap vectors to create its two transmitted bits. This encoder is thus nonsystematic and nonrecursive. Because of their added
complexity, non-systematic codes can have error-correct abilities superior to those of SN codes with the same constraint length. SR is similar to the NN encode except it has feedback and one output bit is identical to the input bit.

In this paper, we use non-systematic and non-recursive convolutional codes because of their error-correction abilities. The performance analysis of convolutional codes is based on trellis diagram corresponding to the generator polynomial. In conventional convolutional coding, only one initial state is available for trellis contrast to convolutional SLM coding. In convolutional SLM coding, n PMEPR control bits are added to spread data $S_k$ followed by an interleaver, which enables $U=2^n$ different initial states for the trellis and result in $U$ different candidates, all associated to $S_k$.

B. Cyclic SLM Codes [19]

Cyclic codes are a sub-class of linear block codes which is easy to encode and decode. Any cyclically shifted version of a cyclic codeword is another codeword. If $U=(u_0,u_1,u_2...,u_{n-1})$ is a codeword then an end-round cyclic shift $U^{(i)}=(u_{n-i},u_{n-i+1},...,u_{n-1},u_0,u_1,...,u_{n-i-1})$ is also a codeword.

In general, $U^{(i)}=(u_{n-i},u_{n-i+1},...,u_{n-1},u_0,u_1,...,u_{n-i-1})$ obtained by $i$ end-around cyclic shifts is a codeword. It is convenient to represent the cyclic codeword by a polynomial with coefficients equal to the components of the codeword:

$$U(X) = u_0 + u_1X + u_2X^2 + ... + u_{n-1}X^{n-1}$$

(6)

1) Generator polynomial for cyclic codes

An $(n,k)$ cyclic code, where $n$ and $k$ are the codeword length and message length respectively, is described by a generator polynomial:

$$g(X) = [1 + g_1X + g_2X^2 + ... + g_{n-k}X^{n-k}]$$

(7)

Note that $g_0 = g_{n-k} = 1$ always hold.

Message as a polynomial:

$$m(X) = [m_0 + m_1X + m_2X^2 + ... + m_{k-1}X^{k-1}]$$

(8)

The codeword polynomial is

$$U(X) = m(X)g(X)$$

(9)

An $(n,k)$ cyclic code generator polynomial $g(X)$ must be degree $(n-k)$, and a factor on $X^n+1$, i.e.

$$X^{n+1} = g(X)h(X)$$

(10)

where $h(X)$ is also a generator which generates $(n,n-k)$ cyclic code.

2) Encoding in systematic form

Given a message polynomial

$$m(X) = [m_0 + m_1X + m_2X^2 + ... + m_{k-1}X^{k-1}]$$

Multiplying message polynomial by $X^{n-k}$, we have

$$X^{n-k}m(X) = [m_0X^{n-k} + m_1X^{n-k+1} + ... + m_{k-1}X^{n-1}]$$

(11)

Divide Eq.(11) by $g(X)$, the result can be rewritten as

$$X^{n-k}m(X) = q(X)g(X) + p(X)$$

(12)

where the remainder can be written as

$$p(X) = p_0 + p_1X + ... + p_{n-k-1}X^{n-k-1}$$

(13)

In other words $p(X) = X^{n-k}m(X) \mod g(X)$.

Adding $p(X)$ on both sides of Eq. (11), we have

$$p(X) = X^{n-k}m(X) = q(X)g(X) = U(X).$$

(14)

The left-hand side is recognized as the valid codeword polynomial since it is a multiple of $g(X)$ it has the message embedded in it, and it has a degree of $n-k$ or less. This code polynomial is in systematic form since the first $n-k$ bits are the parity bits and last $k$ bits are the message bits.

$$U = (p_0, p_1, ..., p_{n-k-1}, m_0, m_1, ..., m_{k-1})$$

(15)

C. LDPC SLM Codes [19], [20]

LDPC codes are block codes with parity-check matrices that contain only a very small number of non-zero entries. It is the sparseness of parity check matrix $H$ which guarantees both a decoding complexity which increases only linearly with the code length and a minimum distance which also increases linearly with the code length. Aside from the requirement that $H$ be sparse, an LDPC code itself is no different to any other block code. Indeed existing block codes can be successfully used with the LDPC iterative decoding algorithms if they can be represented by a sparse parity-check matrix. Generally, however, finding a sparse parity-check matrix for an existing code is not practical. Instead LDPC codes are designed by constructing a sparse parity-check matrix first and then determining a generator matrix for the code afterwards. The biggest difference between LDPC codes and classical block codes is how they are decoded. Classical block codes are generally decoded with Maximum-Likelihood (ML) like decoding algorithms and so are usually short and designed algebraically to make this task less complex. LDPC codes however are decoded iteratively using a graphical representation of their parity-check matrix and thus are designed with the properties of $H$ as a focus.

An LDPC code parity-check matrix is called $(w_c,w_r)$ regular if each code bit is contained in a fixed number, $w_c$, of parity checks and each parity-check equation contains a fixed number, $w_r$, of code bits.

$$H = \begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 & 1
\end{bmatrix}$$

Above $H$ matrix is an example of regular LDPC parity-check matrix with $w_c=2$ and $w_r=3$.

In this paper an irregular systematic LDPC code is used, with code word length $n=10$ and message length $k=5$, having following parity-check matrix.

$$H = \begin{bmatrix}
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

168
A generator matrix for a code with parity-check matrix \( H \) can be found by performing Gauss-Jordan elimination on \( H \) to obtain it in the form
\[
H = [A, I_{n-k}]
\]
(16)
where \( A \) is an \((n-k) \times k\) binary matrix and \( I_{n-k} \) is the identity matrix of order \( n-k \). The generator matrix is then
\[
G = [I_k, A^T].
\]

The row space of \( G \) is orthogonal to \( H \). Thus if \( G \) is the generator matrix for a code with parity-check matrix \( H \), then we obtain \( GH^T = 0 \).

**VII. SIMULATIONS AND RESULT DISCUSSION**

To verify the effectiveness of the proposed method, some simulations are proposed. In experiment the simulation parameters are chosen as in Table I.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Sub-carriers</td>
<td>128</td>
</tr>
<tr>
<td>Spreading Factor</td>
<td>8</td>
</tr>
<tr>
<td>Spreading Sequence</td>
<td>Walsh-Hadamard</td>
</tr>
<tr>
<td>Convolutional code generator polynomial</td>
<td>((133,171)_n)</td>
</tr>
<tr>
<td>Convolutional code constraint length</td>
<td>7</td>
</tr>
<tr>
<td>Cyclic code generator polynomial</td>
<td>(1+X^2+X^3)</td>
</tr>
</tbody>
</table>

The CCDF of the PMEPR using NN convolutional code in PIEC-SLM method for the PMEPR reduction in the MC-CDMA system are shown in the Fig. 4 and Fig. 5. It is observed from the CCDF in Fig. 4 that PIEC-SLM using NN convolutional code with \( n=6 \) PMEPR control bits that results in selection set size of \( U=64 \) reduces PMEPR up to 14dB and 8dB compared to uncoded MC-CDMA signal at \( 10^{-3} \) and \( 10^{-0} \) probabilities respectively. Thus, the NN convolutional code in PIEC-SLM method is an effective way to improve the PMEPR performance of MC-CDMA system.

CCDF with \( n=2, 3, ..., 7 \) PMEPR control bits that result in different selection set size values given as \( U=4, 8, ..., 128 \) using NN convolutional code is shown in Fig. 5. It is observed from the Fig. 5 that the effectiveness of the PIEC-SLM method as it achieve a reduction of 11.5dB with \( n=2 \) PMEPR control bits at \( 10^{-3} \) probability. It is also observed from Fig. 5 that PMEPR reduction increases as the number of PMEPR control bits increases as in Fig. 5, a PMEPR reduction of 12dB with \( n=7 \) PMEPR control bits at probability of \( 10^{-3} \) is observed.

The CCDF of the PMEPR using irregular systematic cyclic code in PIEC-SLM method for the PMEPR reduction in the MC-CDMA system are shown in the Fig. 6 and Fig. 7. It is observed from the CCDF in Fig. 6 that PIEC-SLM using systematic cyclic code with \( n=6 \) PMEPR control bits that results in selection set size of \( U=64 \) reduces PMEPR up to 11dB and 8dB compared to uncoded MC-CDMA signal at \( 10^{-3} \) and \( 10^{-0} \) probabilities respectively. Thus, the systematic cyclic code in PIEC-SLM method is an effective way to improve the PMEPR performance of MC-CDMA system.

The CCDF of the PMEPR using irregular systematic
LDPC code in PIEC-SLM method for the PMEPR reduction in the MC-CDMA system are shown in the Fig. 8 and Fig. 9. It is observed from the CCDF in Fig. 8 that PIEC-SLM using irregular systematic LDPC code with \( n=6 \) PMEPR control bits that results in selection set size of \( U=64 \) reduces PMEPR up to 11dB and 8dB compared to uncoded MC-CDMA signal at \( 10^{-3} \) and \( 10^{-6} \) probabilities respectively.

![Fig. 7. CCDF of the PMEPR of systematic cyclic SLM.](image)

Thus, the irregular systematic LDPC code in PIEC-SLM method is an effective way to improve the PMEPR performance of MC-CDMA system.

CCDF with \( n=2, 3, 4, 5 \) PMEPR control bits that result in different selection set size values given as \( U=4, 8, 16, 32 \) using irregular systematic LDPC code is shown in Fig. 9. It is observed from the Fig. 9 that the effectiveness of the PIEC-SLM method as it achieve a reduction of 6dB with \( n=2 \) PMEPR control bits at \( 10^{-3} \) probability. It is also observed from Fig. 9 that PMEPR reduction increases as the number of PMEPR control bit increases as in Fig. 9, a PMEPR reduction of 10.5 dB with \( n=5 \) PMEPR control bits at probability of \( 10^{-3} \) is observed.

Cyclic code PMEPR performance in PIEC-SLM method for MC-CDMA PMEPR reduction can be improved by using some complex generator polynomials like golay codes and bose-chadhuri-hocquenghem codes. LDPC PMEPR performance in PIEC-SLM method for MC-CDMA PMEPR reduction can be improved by using a sparse matrix like 32400-by-64800 an irregular LDPC code used in digital video broadcasting standard.

![Fig. 9. CCDF of the PMEPR of irregular systematic LDPC SLM.](image)

Fig. 10 shows a comparison among three error control codes used in the PIEC-SLM method. Fig. 10 demonstrates the effectiveness of the PIEC-SLM method and capability of error control codes, used in this paper, to improve PMEPR performance of MC-CDMA signal. Fig. 10 also demonstrates that convolutional code reduces PMEPR the most with respect to other two error control codes used in this paper.

![Fig. 10. CCDF of the PMEPR of convolutional, cyclic and LDPC SLM.](image)

VIII. CONCLUSIONS

In this paper, an improved SLM method for the PMEPR reduction in the MC-CDMA system is presented. The proposed method uses \( n \) PMEPR control bits to generate \( 2^n \) sequences of the original data sequence and each sequence processed by an interleaver and error control code to improve PMEPR performance. Convolutional, cyclic and LDPC error control codes are used in the PIEC-SLM method to reduce the PMEPR. The simulation results verify the effectiveness and versatility of the PIEC-SLM method to improve the PMEPR performance. The simulation results show that the NN convolutional code improves PMEPR performance at most compared to cyclic and LDPC codes. The simulation results also show that a large PMEPR reduction can be achieved by increasing the number of PMEPR control bits. The PIEC-SLM method achieves significant PMEPR reduction for MC-CDMA system without requiring any side information to the receiver to recover the original signal but with little complexity.
ACKNOWLEDGEMENT

This work was partly supported by the National Natural Science Foundation of China (No.61172110, No.61172107); Specialized Research Fund for the Doctoral Program of Higher Education of China (200804100015); Dalian Municipal Science and Technology Fund Scheme of China (No.2008J23JH025); the Fundamental Research Funds for the Central Universities of China (DUT13LAB06).

REFERENCES