# Control Chua Chaos via Optimal Generalized Backstepping Method

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*Abstract*—**In this article, first famous chaotic system, Chua equation, was choiced as chaotic system. One of the best control methods that would be used for stabilization this systems, was Backstepping. In this article this technique is improved to Generalized Backstepping Method (GBM). For this new method, exhibit a new theorem and its proof and for showing its abilities, control Chua equation. Generalized Backstepping approach consists of parameters which accept positive values. The system replied differently for each value. Genetic algorithm can select suitable and optimal values for the parameters. GA by minimizing the fitness function can find the optimal values for the parameters. Fitness function forces the system error to decay to zero rapidly that it causes the system to have a short and optimal setting time. Fitness function also makes an optimal controller and causes overshoot to reach to its minimum value. This hybrid makes an optimal backstepping controller.**

*Index Terms*—**System, lyapunov, generalized backstepping method, genetic algorithm.**

## I. Introduction

One of the most important phenomenons in some systems is chaos; so control chaotic systems is hard and shows the abilities of control techniques. Chaos control and synchronization have been intensively studied during the last decade [1]–[5]. Recently, backstepping method has been applied to chaos control and synchronization successfully [6]–[8]. The Backstepping Method (BM) couldn't obtain good performance in non strict-feedback nonlinear systems and also in some MIMO nonlinear systems. Generalized Backstepping Method (GBM) is introduced in this article [9]. This technique is called GBM because of its similarity to Backstepp ing and more applications in systems than it; Backstepping method is used only to strictly feedback systems but GBM expand this class.. The GBM could have control cost lower than BM. The main contribution of this article is optimizing the GBM with Genetic Algorithm (GA). Genetic algorithm optimizes the controller to gain optimal and suitable values for the parameters. GA minimizes the fitness function to find minimum current value. On the other hand fitness function finds minimum value to minimize least square errors.

The paper is organized as follows. Section 2 describes The Generalized Backstopping Method. Section 3 presents Genetic Algorithms. Section4 describes Controlling Chua System. In section 5 the overall discussion of the simulation results for different systems presented. Section 6 provides conclusion of the study

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## II. The Generalized Backstepping Method

Generalized Backstepping method will be applied to a certain class of autonomous nonlinear systems which are expressed as follow.

$$\begin{cases} \dot{X} = F(X) + G(X)\eta \\ \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u \end{cases} \tag{1}$$

In which $\eta \in \Re$ and $X = [x_1, x_2, ..., x_n] \in \Re^n$. In order to obtain an approach to control these systems, we may need to prove a new theorem as follow.

**Theorem:** Suppose Equationuation.1 is available, then suppose the scalar function $\Phi_i(x)$ for the $i_{th}$ state could be determined in a manner which by inserting the $i_{th}$ term for $\eta$, the function $v(x)$ would be a positive definite equation 3 with negative definite derivative.

$$V(X) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \tag{2}$$

Therefore, the control signal and also the general control Lyapunov function of this system can be obtained by equation 3,4.

$$u = \frac{1}{g_0(X,\eta)}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\Phi_i}{\partial x_j}\left[f_j(X)+g_j(X)\eta\right] - \sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i\left[\eta - \Phi_i(X)\right] - f_0(X,\eta)\right\} \tag{3}$$

$$k_i > 0, i = 1,2,...,n$$

$$V_t(X,\eta) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n}\left[\eta - \Phi_i(X)\right]^2 \tag{4}$$

**Proof:** The Equation 1 can be represented as the extended form of equation 5.

$$\begin{cases} \dot{x}_i = f_i(X) + g_i(X)\eta; i = 1,2,...,n \\ \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u \end{cases} \tag{5}$$

$v(x)$ is always positive definite and therefore the negative definite of its derivative should be examined; it means $w(x)$ in Equation 6 should always be positive definite, so that $\dot{V}(X)$ would be negative definite.

$$\dot{V}(X)=\sum_{i=1}^{n}x_i\dot{x}_i=\sum_{i=1}^{n}x_i\left[f_i(X)+g_i(X)\Phi_i(X)\right]\le -W(X) \tag{6}$$

By $u_0=f_0(X,\eta)+g_0(X,\eta)u$ and adding and subtracting $g_i(X)\Phi_i(X)$ to the $i_{th}$ term of equation 5 and 7 would be obtained.

$$\begin{cases}\dot{x}_i=\left[f_i(X)+g_i(X)\Phi_i(X)\right]+g_i(X)\left[\eta-\Phi_i(X)\right]\\ \dot{\eta}=u_0 \qquad i=1,2,\ldots,n\end{cases} \tag{7}$$

Now we use the following change of variable.

$$z_i=\eta-\Phi_i(X)\Rightarrow\dot{z}_i=u_0-\dot{\Phi}_i(X) \tag{8}$$

$$\dot{\Phi}_i(X)=\sum_{j=1}^{n}\frac{\partial\Phi_i}{\partial x_j}\left[f_j(X)+g_j(X)\eta\right] \tag{9}$$

Therefore, the equation 7 would be obtained as follows:

$$\begin{cases}\dot{x}_i=\left[f_i(X)+g_i(X)\Phi_i(X)\right]+g_i(X)\left[\eta-\Phi_i(X)\right]\\ \dot{z}_i=u_0-\dot{\Phi}_i \qquad ;i=1,2,\ldots,n\end{cases} \tag{10}$$

Regarding that $z_i$ has n states, the $u_0$ can be considered with n terms, provided that the equation 11 would be established as follows.

$$u_0=\sum_{i=1}^{n}u_i \tag{11}$$

Therefore, the last term of Equation 10 would be converted to equation 12.

$$\dot{z}_i=u_i-\dot{\Phi}_i(X)=\lambda_i \tag{12}$$

At this Stage, the control Lyapunov function would be considered as equation 13.

$$V_t(X,\eta)=\frac{1}{2}\sum_{i=1}^{n}x_i^2+\frac{1}{2}\sum_{i=1}^{n}z_i^2 \tag{13}$$

This is a positive definite function. Now it is sufficient to examine negative definitely of its derivative.

$$\dot{V}_t(X,\eta)=\sum_{i=1}^{n}\frac{\partial V(X)}{\partial x_i}\left[f_i(X)+g_i(X)\Phi_i(X)\right]+\sum_{i=1}^{n}\frac{\partial V(X)}{\partial x_i}g_i(X)+\sum_{i=1}^{n}z_i\lambda_i \tag{14}$$

In order that the function $\dot{V}_t(X,\eta)$ would be negative definite, it is sufficient that the value of $\lambda_i$ would be selected as the equation 15.

$$\lambda_i=-\frac{\partial V(X)}{\partial x_i}g_i(X)-k_iz_i \qquad ;k_i>0 \tag{15}$$

Therefore, the value of would be obtained from following equation.

$$\dot{V}_t(X,\eta)=\sum_{i=1}^{n}x_i\left[f_i(X)+g_i(X)\Phi_i(X)\right]-\sum_{i=1}^{n}k_iz_i^2\le -W(X)-\sum_{i=1}^{n}k_iz_i^2 \tag{16}$$

Which indicates that the negative definitely status of the function $\dot{V}_t(X,\eta)$. Consequently, the control signal function, using the equation 7, 9 and 11 would be converted to 17.

$$u_0=\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\Phi_i}{\partial x_j}\left[f_j(X)+g_j(X)\eta\right]-\sum_{i=1}^{n}x_ig_i(X)-\sum_{i=1}^{n}k_i\left[\eta-\Phi_i(X)\right] \tag{17}$$

Therefore, using the variations of the variables which we carried out, the equation 3, 4 can be obtained. Now, considering the unlimited region of positive definitely of $V_t(X,\eta)$ and negative definitely of $\dot{V}_t(X,\eta)$ and the radially unbounded space of its states, global stability gives the proof.

## III. Genetic Algorithm

The most of optimization algorithms are based on the gradient of the cost function, so for the ill choice of the initial point or the interval search. these algorithms can be misled on the locally optimum and can't achieve the globally optimum . For this problem, a class of optimization algorithm, like genetic algorithms, is developed to avoid this constraint.

In its most general usage, Genetic algorithms refer to a family of computational models inspired by evolution. These algorithms start with many initial point in order to cover all search interval and encode a potential solution to a specific problem on a simple chromosome like data structure and apply recombination operators to these structures so as to preserve critical information. An implantation of genetic algorithms begins with a population of chromosomes randomly bred. We evaluate each chromosome by using the objective function called Fitness function. In order to apply the genetic reproductive operations called crossover and mutation, we select, randomly, two individuals called parents and we apply the crossover operation, if its probability reaches, between parents by exchanging some of their bits to produce two children . A mutation is the second operator applied on the single children by inverting its bit if the probability reaches. After this stage we obtain two population : a parent population and a children population, the individual who has a good solution is preserved [10].

The genetic algorithms are used to search the optimal parameter k ($k_j$ j=1,2 is positive constant) in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response. The fitness function used is

$$f=\frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(x_i-x_{di}\right)^2} \tag{18}$$

$x_i$ is system state and $x_{di}$ is favorit mood for $x_i$. based the system purpose for placing the states at zero value. $x_{di}$ is eaqual with zero.

TABLE I: SHOW THE GENETIC PARAMETERS

| Parameters | Values |
|---|---|
| Size population | 100 |
| Maximum of generation | 300 |
| Prob.crossover | 0.75 |
| Prob.mutation | 0.001 |
| $K_j$ search interval de | [0.1 10] |

## IV. CONTROLLING CHUA SYSTEM

Consider the Chua's system

$$\begin{aligned} \dot{x} &= -\beta y \\ \dot{y} &= z - y + x \\ \dot{z} &= \alpha(y - z^3 - \gamma z) \end{aligned} \tag{19}$$

where $\alpha = 10, \beta = 16, \gamma = -0.143$ are circuit parameters and $x, y, z$ are state variables and initial condition (x,y,z)=(0.003,0.001,0.002). The nonlinear chua's system given by equation (19) exhibits varieties of dynamical behaviour including chaotic motion displayed in "Fig.1", and "Fig.2".

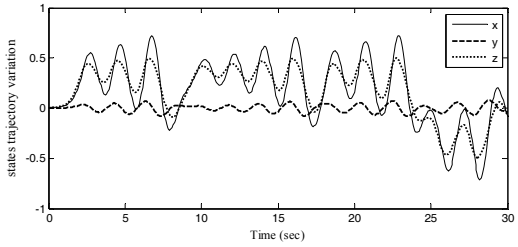



Fig. 1. show the states trajectory variation for chua's system

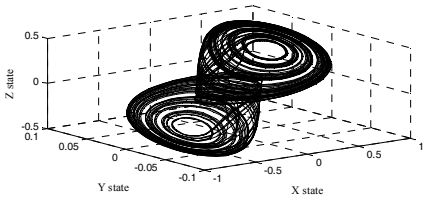



Fig. 2. show the phase portrait of chua's system

In order to convert chua's system equations to the general state of Eq.1 , the change variable $r = y - x$ should be carried out and introduce control signal $u(t)$. Therefore, the Eq.19 would be converted to Eq.20 , as follows.

$$\begin{aligned} \dot{r} &= z - r + \beta y \\ \dot{y} &= z - r \\ \dot{z} &= \alpha(y - z^3 - \gamma z) + u \end{aligned} \tag{20}$$

**Stabilization of the state:** In order to use the theorem, it is sufficient to establish Eq.21,22

$$\phi_1(r, y) = -\beta y \tag{21}$$

$$\phi_2(r, y) = r - y \tag{22}$$

According to the theorem, the control signal will be obtained from the Eq.23

$$u = \dot{r} - (\beta + 1)\dot{y} - r - y - k_1(z - \varphi_1) - k_2(z - \varphi_2) - \alpha(y - z^3 - \gamma z) \tag{23}$$

After using the Genetic Algorithm obtained these optimal parameters : $k_1 = 3.334, k_2 = 0.335$ . The result estimated are showed in "Fig.3", to "Fig.6". The states trajectory variation for chua's system are showed in "Fig.3", to "Fig.5". The control signal trajectory variation is showed in "Fig.6".

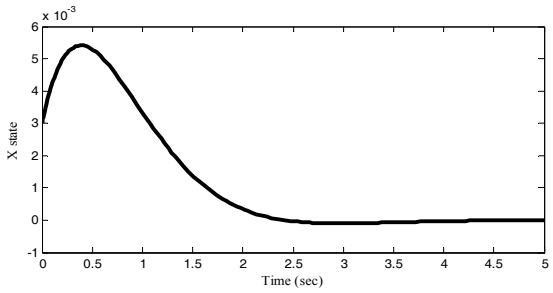



Fig. 3. show the x state trajectory variation for chua's system

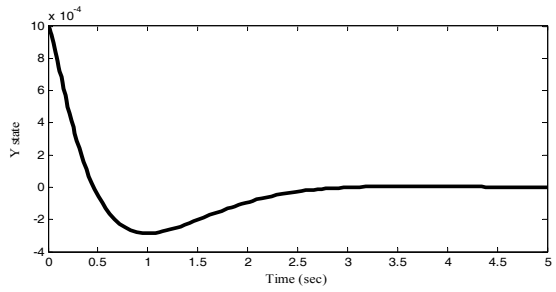



Fig. 4. show the y state trajectory variation for chua's system

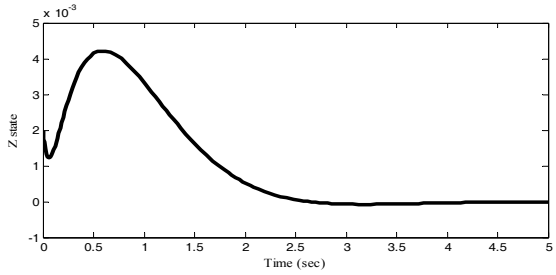



Fig. 5. show the z state trajectory variation for chua's system

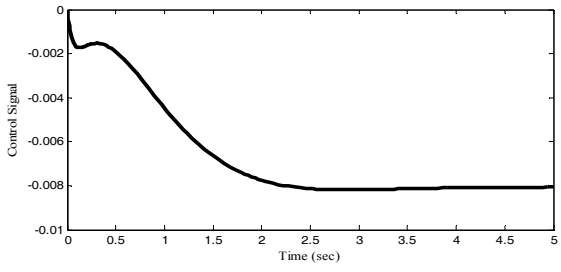



Fig. 6. show the control signal trajectory variation for chua's system

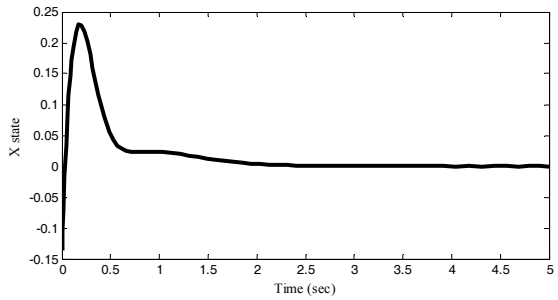


Fig. 7. show the x state trajectory variation for chua's system

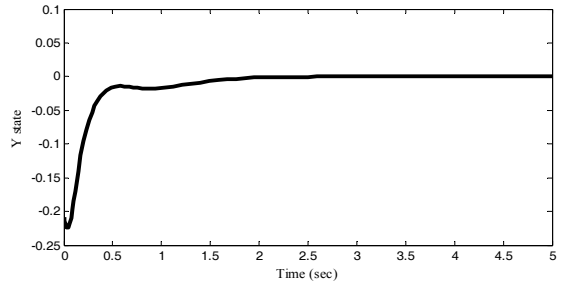


Fig. 8. show the y state trajectory variation for chua's system

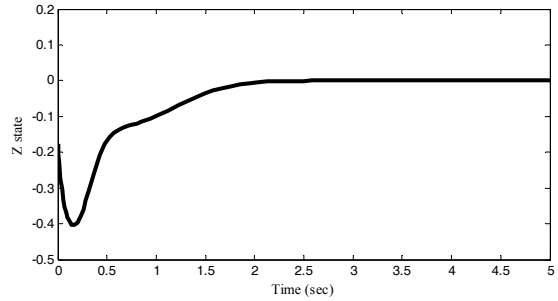


Fig. 9. show the z state trajectory variation for chua's system

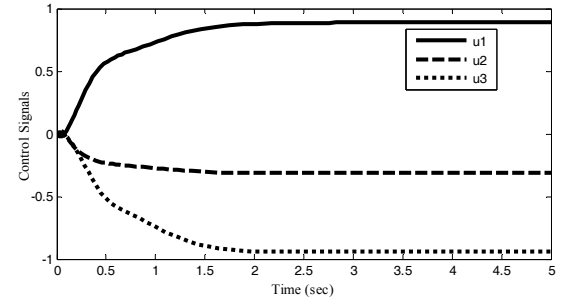


Fig. 10. show the control signals trajectory variation for chua's system

## V. Discussion

In ref.[11] the Backstepping method was used to control the chua's system that Fig.7 to Fig.10 represent the simulation results, But in this study is used Generalized Backstepping method. Now we would compare the results of the proposed method and the results in ref.[11]. The result of ref.[11] are showed in "Fig.7", to "Fig.10". The states trajectory variation for chua's system are showed in "Fig.7", to "Fig.9". The control signals trajectory variation is showed in "Fig.10".

By comparing the Figs., the following results can be obtained.

- In the Generalized Backstepping Method in relation to the Backstepping Method [11], the system states are stabilized by a more limited control signal. Consequently, it is less possible that the control signal to be saturated.
- In the Generalized Backstepping Method inrelation to the Backstepping Method [11], Control will be accomplished in a much shorter overshoot.

Considering the results obtained from simulations, the much more efficiency of Generalized Backstepping Method in relation to the Backstepping Method will be demonstrated.

## VI. Conclusion

In this study, a new method to control nonlinear systems is presented. The proposed method which is called Generalized Backstepping Method, by feed backing the dynamics of system and without eliminating the nonlinear dynamics, a controller is designed. A theorem is expressed for this method and the proof is given. Consequently, using this method, a controller is designed for the Chua chaotic system which is compared with the results obtained from the controller using the Backstepping Method.

The designed controller consists of parameters which accept positive values. The controlled system presents different behavior for different values. Improper selection of the parameters causes an improper behavior which may cause serious problems such as instability of system. Genetic algorithm optimizes the controller to gain optimal and proper values for the parameters. For this reason GA minimize the fitness function to find minimum current value for it. On the other hand fitness function finds minimum value to minimize least square errors. By this approach the setting time and overshoot reach to their minimum values that are demonstrated to have more optimal values when compared with previous methods.

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