Application of Coevolutionary Algorithm for Wavelet Filter Design

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Abstract—Designing wavelets using analytic methods often needs to solve complicated systems of equations that cannot be solved easily. This paper proposed a new wavelet filter design methods using a coevolutionary algorithm. Each wavelet is made up of low and high-pass finite impulse response (FIR) filters so we separate our design into two parts: the first part produce low-pass high resolution filter using GA with a two parameters fitness function that minimize the Uncertainty rectangle diameter and low-pass bandwidth, in the second part we take some of the best individuals from the first population in each generation and combine them with population of the second part and generate a new population then find best individuals using the fitness function which is derived from Heisenberg Uncertainty Principle and moment cancellation property. Then combine these new individuals with the population of the first group and repeat the optimization using GA and the new generated population. With these methods we optimize two filters in a parallel way. The reason that we cannot optimize filters independently is that uncertainty rectangle diameters in a function of both of the filters.

***Index Terms*—Evolutionary algorithm, filter design, wavelet**

I. Introduction

A. Wavelets

The Fourier transform has reshaped the in which engineers interpret signals but it is evident that by breaking a signal into a series of trigonometric basis functions time-varying features can not be captured because Fourier basis functions are localized in frequency but not in time. Gabor introduced one alternative to localize the Fourier transform though short time Fourier transform (STFT). However the constraint of Heisenberg uncertainty principle that limits obtainable resolution considerably prompting an alternative approach to time-frequency analysis featuring basis functions that have compact support in both frequency and time to yield a multi-resolution analysis termed the Wavelet transform [1].

Nowadays Wavelets are powerful signal processing tool that have found applications in a broad spectrum of scientific and applied engineering problems. Some of the current engineering applications include image processing, communication, data storage and compression as well as information extraction for pattern recognition and diagnostics. The time-frequency character of Wavelet transform allows adaptation of both traditional time and frequency domain system identification approaches to examine nonlinear and nonstationary data. The Discrete Wavelet Transform (DFT) is obtained by repeated filtering and subsampling into two bands with low and high-pass finite impulse response (FIR) and the inverse process gives perfect reconstruction if the wavelet is orthonormal. So designing appropriate filters that have some important characteristics is the main part of wavelet analysis. We show these characteristics by some conditions and introduce a function that minimization of it means satisfaction of the conditions. There are many methods for minimizing functions, in this paper Genetic Algorithm (GA) and coevolutionary methods is used for this purpose because using analytic methods for this case leads to complicated equations that cannot be solved easily.

B. Genetic Algorithms

Genetic Algorithms are globally stochastic search techniques that emulate the laws of evolution and genetics to try to find optimal solution to complex optimization problems. GA's are theoretically and empirically proven to provide robust search in complex spaces and they are widely applied in engineering, business and scientific circles [2].

GA's are different from more normal optimization and search procedures in four ways [3]:

●GA's work with a coding of a parameter set not the parameters themselves.

●GA's search from a population of points not a single point.

●GA's use objective function information not derivatives or the auxiliary knowledge but with modifications they can exploit analytical gradient information if it's available.

●GA's use probabilistic transition rules not deterministic rules.

A simple genetic algorithm that yields good results in many practical problems consists of three genetic operators:

●REPROUCTION is a process in which individual strings are copied according o their objective or fitness function. In general, operator of reproduction guarantees survival of the better individuals to the next generation with a higher probability, which is an artificial version of natural selection.

●CROSSOVER is a partial exchange of the genetic content between couples of members of the population. This task can be done in several different ways and it also depends on the representation schemes chosen.

●MUTATION is needed because even through reproduction and crossover effectively search and recombine extant notions occasionally they may become overzealous and lose some potentially useful genetic materials. In GA's the mutation operation protects against such an irrecoverable loss. In other words mutation tries to escape from a local maximum or minimum of the fitness function.

The flowchart of a simple genetic algorithm can be seen in Fig. 1.

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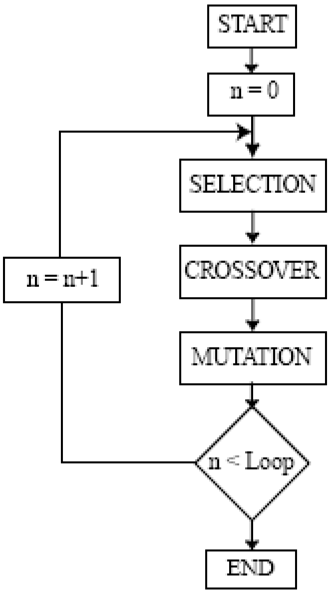


Fig. 1. The fiow chat of a simple genetic algorithm.

C. Coevolutionary Algorithm

Coevolutionary algorithms are base on multiple parallel GA optimizations that have interconnection with each other. These methods help us to optimize different genotypes simultaneously that cause to reach the solution in fewer generations. The other advantage of this method is that in simple GA's tuning fitness function coefficients has many problems because inappropriate coefficients may cause slower optimization or bad results. In our method optimizations of high-pass and low-pass filter are done separately but the high-pass and low-pass filters have important effects on resolution of wavelet. The population size of the first GA is 40 we take 8 of the best individuals and combine them with all of the twenty individuals of the second one. And compare these 80 generated individuals with the second fitness function and take 8 of them to produce the next generation.

The best individuals chosen in each generation is fed as initial population to Matlab GA toolbox after combination with individuals of other population.

II. Backgrounds

Before we propose the fitness functions we should describe some wavelets characteristic backgrounds used in the function.

A. Wavelet Moments

Wavelets Kth moments of Φ(t) and Ψ(t) are defined as:

$$m(k) = \int t^k.\phi(t).dt \tag{1}$$

and

$$m_1(k) = \int t^k.\psi(t).dt \tag{2}$$

And the discrete moments of h[n] and h1[n] as:

$$\mu(k) = \sum_n n^k.h[n] \tag{3}$$

and

$$\mu_1(k) = \sum_n n^k.h_1[k] \tag{4}$$

where h[n] and h1[n] are low- and high-pass FIR filters coefficients respectively.

Wavelets exists only if a regularity condition is satisfied:

$$\sum h(n) = \sqrt{2} \tag{5}$$

Requiring moments to be zero has several interesting consequences. There are some methods for designing wavelets with the maximum number of vanishing moments such as Daubechies method introduced in [4].

B. Heisenberg Uncertainty Principle [5]

The lower limit on the time-frequency resolution that can be obtained with wavelet transforms in theoretically:

$$\Delta\omega.\Delta t \geq \frac{1}{2} \tag{6}$$

where $\Delta\omega$ and Δt are defined as:

$$\Delta t^2 = \frac{\int \tau^2.f(\tau).d\tau}{\int f(\tau)^2.d\tau}$$
$$\tau = t - \bar{t} \tag{7}$$

and

$$\Delta\omega^2 = \frac{\int \omega^2.\left|F(\omega)\right|^2.d\omega}{\int \left|F(\omega)\right|^2.d\omega} \tag{8}$$

This states that the time and frequency localization of information cannot simultaneously be arbitrarily small. It can be shown that the continuous, infinite tine, Gabor wavelet is the only wavelet that achieves the minimum. The lower the Heisenberg uncertainty the better is the resolving power of wavelet. The uncertainty is invariant over the scales used in DWT; as data in filtered and subsampled the time resolution is successively halved and the frequency resolution is doubled preserving their product $\Delta\omega.\Delta t$.

We can compute bandwidth $\Delta\omega$ and time dispersion Δt directly for a FIR filter from its coefficients:

$$\Delta\omega^2 = \frac{\pi^2}{3} + 4.\sum_{n=0}^{l-2}\sum_{m=n+1}^{l-1}\frac{(-1)^{m-n}}{P.(m-n)^2}.h[m].h_1[n]$$

where $P = \sum_{n=0}^{l-1} h[n]^2$.

$$\Delta t^2 = \sum_{n=0}^{l-1}(n-\bar{t})^2.h[n]^2$$

where
$$\bar{t} = \frac{\sum n.h[n]}{\sum h[n]} \tag{9}$$

C. Low-Pass Condition

Here we propose a new metric, which has to be minimized to result a low-pass filter. Consider a low-pass filter with cutoff frequency ω_c and C, which is defined as:

$$C = \frac{\int_{\omega_c}^{\pi} |H_1(\omega)|^2 .d\omega}{\int_{0}^{\omega_c} |H_1(\omega)|^2 .d\omega} \tag{10}$$

where $H_1(\omega)$ is the frequency response of the FIR filter. It is obvious that by minimizing C we can obtain a high-pass filter.

D. Fitness Function

Now we can define our fitness functions.

The first GA's fitness function is defined as:

$$J_1(h) = \alpha.|\Delta\omega.\Delta t - .5| + \gamma.C \tag{11}$$

As it can be seen minimizing J1 cause to generate a low-pass filter with minimum uncertainty. γ must be greater than αi when being high-pass is more important than uncertainty minimization. The fitness function of the second part of design can be defined as:

$$J_2(h_1) = \sum \alpha_i.\mu_1(i) + \beta.|\Delta\omega.\Delta t - .5| \tag{12}$$

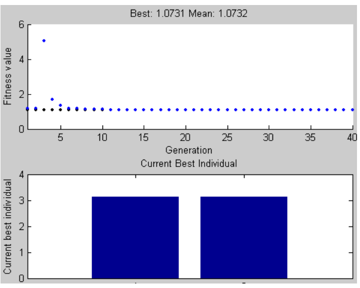
But before we insert the filter coefficients to the fitness function we should multiply them by $\frac{\sqrt{2}}{\sum h[n]}$ to satisfy the wavelet existence condition (5).

In both fitness functions we can replace $\alpha|\Delta\omega.\Delta t - .5|$ with $\alpha\Delta\omega^2+\beta\Delta t^2$ so we can assign β and α independently according to their significance in our design.

E. Design Examples

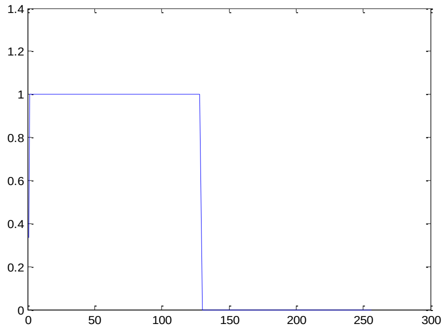
1) Generating FIR Filter with 2 Coefficients:

The first graph of the below figures shows the trajectory of the first fitness function during generations and the second one shows best individuals in the last generation:

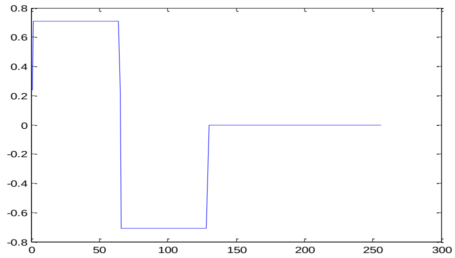


Results:

$$h = [0.7071, 0.7071]$$
$$h_1 = [0.7071, -.07071]$$
$$\Delta t.\Delta\omega = .5679$$



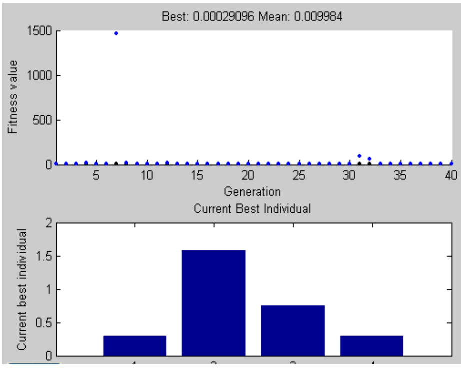
Generated scaling function.



Generated wavelet function.

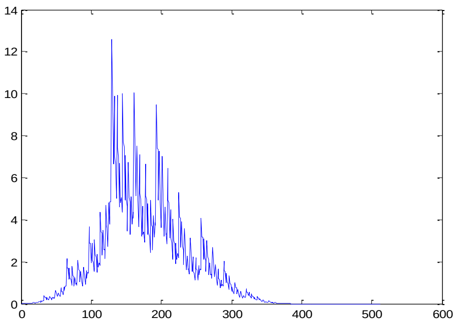
The functions are very similar to Haar Wavelet and Scaling functions.

2) Generating FIR Filter with 4 Coefficients:

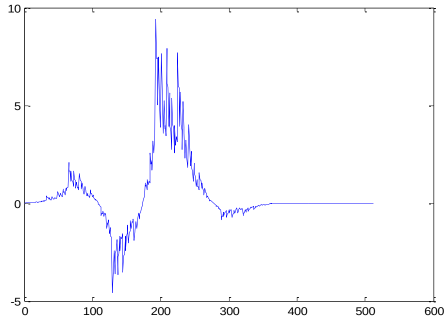


Results:

$$h = [.1395, .7669, .3626, .1452]$$
$$\Delta t^2 = .2235$$
$$\Delta\omega^2 = 1.1878$$
$$\Delta t.\Delta\omega = .5153$$

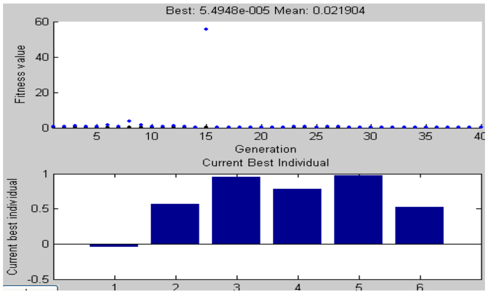


Generated scaling function.



Generated wavelet function.

3) Generating FIR Filter with 6 Coefficients:



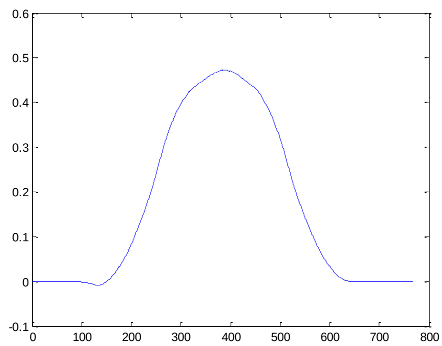
Results:

$$h = [-.0156, .2116, .3591, .2957, .3681, .1953]$$

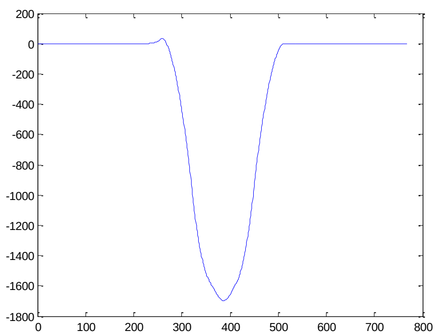
$$\Delta t^2 = .5986$$

$$\Delta \omega^2 = .4261$$

$$\Delta t . \Delta \omega = .5051$$



Generated scaling function.



Generated wavelet function.

Now we compare our wavelets with the ones generated in [5]

L=2	Our Wavelet	MinUnCert2
Δω.Δt	.5679	.568

Table I.

L=4	Our Wavelet	MinUnCert4
Δω	1.0898	1.103
Δt	.4727	.506
Δω.Δt	.5153	.559

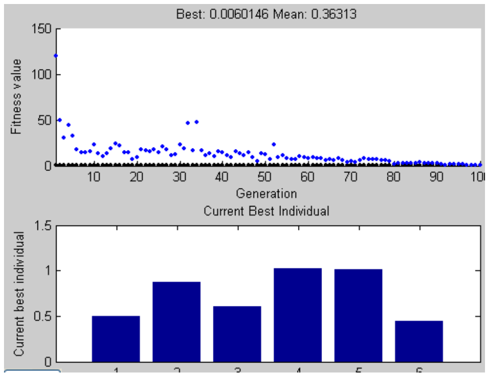
Table II.

L=4	Our Wavelet	MinUnCert6
Δω	.6527	.995
Δt	.7736	.686
Δω.Δt	.5051	.682

Table II.

As it can be seen the resolution of wavelets generated with GA is much better than wavelets generated with the other method.

4) Generating Wavelet with Minimum Moments (L=6):



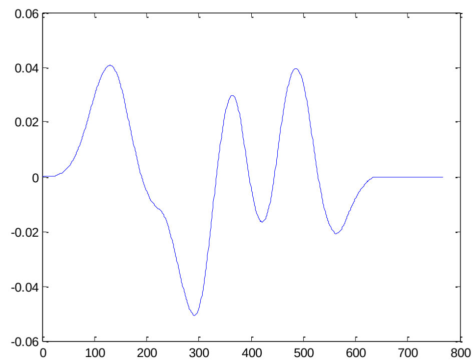
Results:

$$h = [.0740, .2149, .2701, .3035, .3535, .1982]$$

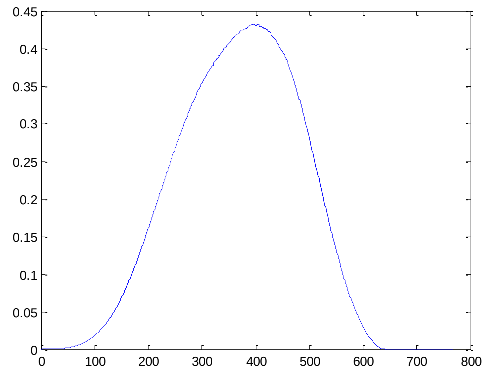
$$h_1 = [.198, -.353, .303, -.270, .214, -.074]$$

$$\Delta \omega . \Delta t = .5132$$

$$\mu_1(0) = .019, \mu_1(1) = -.067, \mu_1(2) = .016$$



Generated wavelet function.



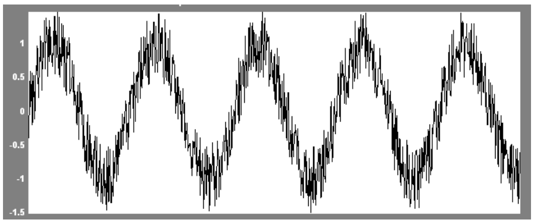
Generated scaling function.

Noise Reduction

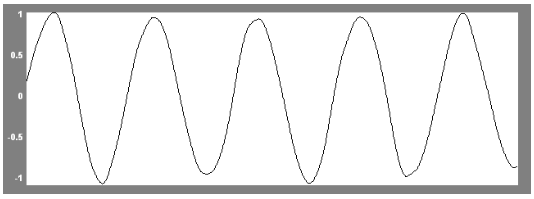
The high-resolution wavelets such as the ones generated above can be used in de-noising signals. We use Matlab Wavemenu de-noising toolbox to show the ability of our wavelet in de-noising problems. First we expand the input signal using our wavelet then use de-noise function.

$$h = [-.0156, .2116, .3591, .2957, .3681, .1953]$$

$$v_{i(t)} = Sin(\omega t + \phi) + n(t)$$



Input signal.



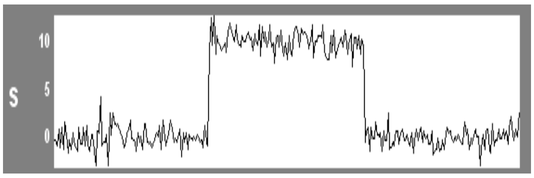
De-noised signal.

Although distortions happened at peak value of the signal but it can be seen that noise was reduced completely.

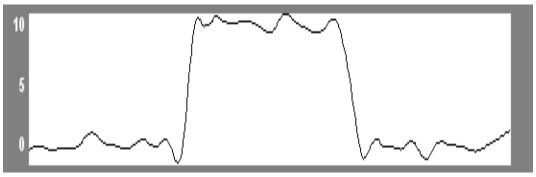
Next example:

$$h = [-.0156, .2116, .3591, .2957, .3681, .1953]$$

$$v_{i(t)} = 10 * u(t - 100) - 10 * u(t - 200) + n(t)$$



Input signal.



De-noised signal.

In this example noise was removed too. The distortion in noise reduction in inevitable in all noise reduction methods so here we have distortion too but the amplitude of distortions tolerable.

These good results show the high ability of wavelets generated with our method in noise reduction.

F. *Concluding Remarks*

In this paper we introduced a wavelet design method using Coevolutionary Algorithm. In some cases such as minimizing uncertainty it results very high-resolution filters but in moment cancellation it can't generate filters with zero moment and some of the moments remain non-zero but the results are much better in comparison with the other paper [5] also it still is a fast and efficient method for designing wavelet indirectly without solving complicated equations. We should consider that Although zero moments is a good specification of wavelet filters but being high-resolution is the most important factor in wavelet designing and most of the wavelet properties depends on it. Another problem of using this method that appears in many of these evolutionary methods is tuning fitness function parameters because they affect results and may lead us to undesired filters. This problem becomes more important when dealing with moment reduction in higher order.

REFERENCES

[1] T. Kijewski and A. Kareem, "Wavelet Transforms for System Identification in Civil Engineering," *Computer-Aided Civil and Infrastructure Engineering*.

[2] Y. Chen, *Hybrid Soft Computing Approach to Identification And Control of Nonlinear Systems*, 2001.

[3] D. E. Goldberg, *Genetic Algorithms in Search Optimization And Machine Learning*, Addison-Wesley, 1989.

[4] C. Sidney Burrus, *Introduction to Wavelets and Wavelet Transform*, Prentice-Hall, 1998.

[5] D. M. Monro, B. E. Bassil, and G. J. Dickson "Orthonormal Wavelets With Balanced Uncertainty," in *Proceedings IEEE International Confrerence on Image Processing ICIP-96*, vol. 2, pp. 581-584