Generalized Backstepping Method Based Control Chen Chaotic System Using Adaptive Neuro-Fuzzy Inference System

A. R. Sahab and M. Taleb Ziabari

***Abstract*—In this paper, first known chaotic system, Chen equation, was selected as chaotic system. One of the best control methods that would be used for stabilization this systems, was Backstepping. In this paper this method is improved to Generalized Backstepping Method (GBM). For this new method, expose a new theorem and its proof and for showing its abilities, control Chen equation. Generalized Backstepping approach consists of parameters which accept positive values.The system responded differently for each value. Genetic algorithm can select appropriate and optimal values for the parameters. GA by minimizing the fitness function can find the optimal values for the parameters. Fitness function forces the system error to decay to zero rapidly that it causes the system to have a short and optimal setting time. Fitness function also makes an optimal controller and causes overshoot to reach to its minimum value. This hybrid makes an optimal backstepping controller.**

***Index Terms*—Chen chaos, lyapunov, generalized backstepping method, ANFIS.**

I. INTRODUCTION

One of the most important phenomenons in some systems is chaos; so control chaotic systems is difficult and shows the abilities of control methods. The Backstepping Method (BM) couldn't achieve good performance in non strict-feedback nonlinear systems and also in some MIMO nonlinear systems. Generalized Backstepping Method (GBM) is introduced in this paper. This method is called GBM because of its similarity to Backstepping and more applications in systems than it; Backstepping method is used only to strictly feedback systems but GBM expand this class. The GBM could have control cost lower than BM. The main contribution of this paper is optimizing the GBM with Genetic Algorithm (GA). Genetic algorithm optimizes the controller to gain optimal and proper values for the parameters. GA minimizes the fitness function to find minimum current value. On the other hand fitness function finds minimum value to minimize least square errors.

In recent years, various techniques and methods have been suggested to obtain chaos control. For example, OGY method [1], feedback and nonfeedback control [2]–[5], inverse optimal control [6]and backstepping designtechnique [7].

Manuscript received May 10, 2012; revised June 10, 2012.

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The paper is organized as follows. Section 2 describes The Generalized Backstopping Method. Section 3 presents Adaptive Neuro-Fuzzy Inference System (ANFIS).Section4 describesControlling Chen System.Section 5 describes tracking any desired trajectory.In section 6 simulation results and discussion of Chen system is represented. In section 7 the overall discussion of the simulation results for different systems presented. Section 8 provides conclusion of the study.

II. THE GENERALIZED BACKSTEPPING METHOD

Generalized Backstepping method will be applied to a certain class of autonomous nonlinear systems which are expressed as follow.

$$\begin{cases} \dot{X} = F(X) + G(X)\eta \\ \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u \end{cases} \tag{1}$$

In which $\eta \in \Re$ and $X = [x_1, x_2, \ldots, x_n] \in \Re^n$. In order to obtain an approach to control these systems, we may need to prove a new theorem as follow.

A. Theorem

Suppose (1) is available, then suppose the scalar function $\Phi_i(x)$ for the i_{th} state could be determined in a manner which by inserting into η, the function $V(x)$ would be a positive definite (3) with negative definite derivative.

$$V(X) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \tag{2}$$

Therefore, the control signal and also the general control Lyapunov function of this system can be obtained by (3) and (4).

$$u = \frac{1}{g_0(X,\eta)}\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\Phi_i}{\partial x_j}[f_j(X) + g_j(X)\eta] - \sum_{i=1}^{n} x_i g_i(X) \\ - \sum_{j=1}^{n} k_i[\eta - \Phi_i(X)] - f_0(X,\eta)\} \ ; \ k_i > 0 \ , \ i = 1,2,\ldots,n \tag{3}$$

$$V_t(X,\eta) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n}[\eta - \Phi_i(X)]^2 \tag{4}$$

B. Proof

(1) can be represented as the extended form of (5).

$$\begin{cases}\dot{x}_i = f_i(X) + g_i(X)\eta \;;\; i = 1,2,...,n \\ \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u\end{cases} \tag{5}$$

$V(X)$ is always positive definite and therefore the negative definite of its derivative should be examined; it means $W(X)$ in (6) should always be positive definite, so $\dot{V}(X)$ would be negative definite.

$$\dot{V}(X) = \sum_{i=1}^{n} x_i \dot{x}_i = \sum_{i=1}^{n} x_i \left[f_i(X) + g_i(X)\Phi_i(X) \right] \tag{6}$$

By $u_0 = f_0(X,\eta) + g_0(X,\eta)u$ and subtracting $\pm g_i(X)\Phi_i(X)$ to the i_{th} term of (5) and (7) would be obtained.

$$\begin{cases}\dot{x}_i = \left[f_i(X) + g_i(X)\Phi_i(X) \right] + g_i(X)\left[\eta - \Phi_i(X)\right] \\ \dot{\eta} = u_0 \qquad\qquad i = 1,2,\ldots,n\end{cases} \tag{7}$$

Now we use the following change of variable.

$$z_i = \eta - \Phi_i(X) \Rightarrow \dot{z}_i = u_0 - \dot{\Phi}_i(X) \tag{8}$$

$$\dot{\Phi}_i(X) = \sum_{j=1}^{n} \frac{\partial \Phi_i}{\partial x_j}\left[f_j(X) + g_j(X)\eta \right] \tag{9}$$

Therefore (7) would be obtained as follows.

$$\begin{cases}\dot{x}_i = \left[f_i(X) + g_i(X)\Phi_i(X) \right] + g_i(X)\left[\eta - \Phi_i(X)\right] \\ \dot{z}_i = u_0 - \dot{\Phi}_i \qquad\qquad i = 1,2,\ldots,n\end{cases} \tag{10}$$

Regarding that z_i has n states, the u_0 can be considered with n terms, provided that (11) would be established as follows.

$$u_0 = \sum_{i=1}^{n} u_i \tag{11}$$

Therefore, the last term of (10) would be converted to (12).

$$\dot{z}_i = u_i - \dot{\Phi}_i(X) = \lambda_i \tag{12}$$

At this Stage, the control Lyapunov function would be considered as (13).

$$V_t(X,\eta) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n} z_i^2 \tag{13}$$

This is a positive definite function. Now it is sufficient to examine negative definitely of its derivative.

$$\begin{aligned}\dot{V}(X,\eta) = &\sum_{i=1}^{n} \frac{\partial V(X)}{\partial x_i}\left[f_i(X) + g_i(X)\Phi_i(X) \right] \\ &+ \sum_{i=1}^{n} \frac{\partial V(X)}{\partial x_i} g_i(X) + \sum_{i=1}^{n} z_i \lambda_i\end{aligned} \tag{14}$$

In order that the function $\dot{V}_t(X,\eta)$ would be negative definite, it is sufficient that the value of λ_i would be selected as (15).

$$\lambda_i = -\frac{\partial V(X)}{\partial x_i} g_i(X) - k_i z_i \quad ; \quad k_i > 0 \tag{15}$$

Therefore, the value of would be obtained from following.

$$\begin{aligned}\dot{V}(X,\eta) = &\sum_{i=1}^{n} x_i \left[f_i(X) + g_i(X)\Phi_i(X) \right] \\ &+ \sum_{i=1}^{n} k_i z_i^2 \le -W(X) - \sum_{i=1}^{n} k_i z_i^2\end{aligned} \tag{16}$$

Which indicates that the negative definitely status of the function $\dot{V}_t(X,\eta)$. Consequently, the control signal function, using (7), (9) and (11) would be converted to (17).

$$\begin{aligned}u_0 = &\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial \Phi_i}{\partial x_j}\left[f_i(X) + g_i(X)\eta \right] \\ &- \sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i\left[\eta - \Phi_i(X)\right]\end{aligned} \tag{17}$$

Therefore, using the variations of the variables which carried out, (3) and (4) can be obtained. Now, considering the unlimited region of positive definitely of $V_t(X,\eta)$ and negative definitely of $\dot{V}_t(X,\eta)$ and the radially unbounded space of its states, global stability gives the proof.

III. Adaptiv Neuro-Fuzzy Inference System (ANFIS)

Adaptive -network-based fuzzy inference system (ANFIS) has been proposed by Jang [8]. The fuzzy inference system is implemented in the framework of adaptive networks using a hybrid learning procedure, whose membership function parameters are tuned using a back propagation algorithm combined with a least square method. ANFIS is capable to deal with uncertainty and imprecision of human knowledge. It has self-organized ability and inductive inference function to learn from the data. ANFIS is a multilayer feed forward network [8]. Each node of the network performs a particular function on incoming signals as well as a set of parameters pertaining to this node. To present the ANFIS architecture, consider two-fuzzy rules based on a first order Sugeno's model [9] shown in Fig. 1.

Rule 1: if (X_1 is A_1) and (X_2 is B_1)
then ($F_1 = P_1X_1 + Q_1X_2 + R_1$)
Rule 2: if (X_1 is A_2) and (X_2 is B_2)
then ($F_2 = P_2X_1 + Q_2X_2 + R_2$)

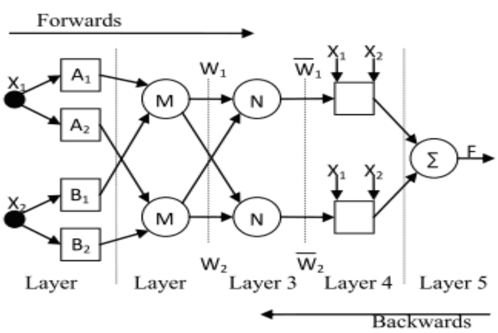

Fig. 1. Simple ANFIS structure

The system has two inputs x_1 and x_2 with one output F. A square node (adaptive node) has parameters and changes during training while a circle node (fixed node) has none. Two membership functions are associated with each input. The rule contains two fuzzy if-Then rules of Takagi and Sugeno's type. The key features of the five layers are described as follows. In the following presentation O_{Li} denotes the output of node 'I' in a layer L [10].

Layer 1: The nodes in this input layer are adaptive and define the membership functions of inputs. The membership function can be bell-shaped or Gaussian. Parameters in this layer are referred to as promise parameters.

$$\begin{aligned} O_{1,i} &= \mu_{Ai}(X_1) \\ O_{1,i} &= \mu_{Bi-2}(X_1) \end{aligned} \;;\; i = 1,2 \tag{18}$$

where A_i and B_i can be any appropriate fuzzy sets in parameter form.

Layer 2: The nodes in this rule layer are fixed. It multiplies all the incoming signals and sends the product out. Output of each node represents the firing strength of a rule

$$O_{2,i} = W_i = \mu_{Ai}(X_1)\mu_{Bi}(X_2) \;;\; i = 1,2 \tag{19}$$

The output of each node is this layer which represents the firing strength of the rule.

Layer 3: The nodes in this normalization layer are fixed. The nodes normalize the firing strengths obtained in Layer 2.

$$O_{3,i} = \overline{W_i} = \frac{W_i}{W_1 + W_2} \;;\; i = 1,2 \tag{20}$$

Layer 4: The nodes in this inference layer are adaptive. The outputs in this layer are the outputs from Layer 3 multiplied by a linear formula. Parameters in this layer are referred to as consequent parameters:

$$O_{4,i} = \overline{W_i}F_i = \overline{W_i}(P_i X_i + Q_i X_2 + R_i) \;;\; i = 1,2 \tag{21}$$

where P_i, Q_i and R_i are designing parameters (consequent parameter since they deal with the then-part of the fuzzy rule).

Layer 5: The nodes in this output layer are fixed. It computes the overall output as the summation of the weighted outputs from Layer 4.

$$O_{5,i} = F = \sum_i \overline{W_i}F_i = \frac{\sum_i W_i F_i}{\sum_i W_i} \;:\; i = 1,2 \tag{22}$$

The ANFIS architecture is not unique. Some layers can be combined and still produce the same output. There are two sets of parameters in the above fuzzy inference system. The overall output is linear in the consequent parameters on layer 3 but nonlinear in the parameters on layer 1. The hybrid learning algorithm detailed in [10] consists of a forward and a backward pass. In the forward pass, the linear parameters are updated using least squares estimator (LSE). In the backward pass, errors of derivatives are calculated for each node starting from the output end and propagating towards the input end of the network. The nonlinear parameters are updated by steepest descent algorithm [10].

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IV. Control Chen System

The Chen system is described by:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= (c - a)x - xz + cy \\ \dot{z} &= xy - bz \end{aligned} \tag{23}$$

where $a = 35, b = 3$ and $c = 28$ are parameters and initial values $(x, y, z) = (10, -10, 10)$. GBM is used to design a controller. In order to control (23) we add a control input u_1 to the 3rd equation of it. Then the controlled system is:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= (c - a)x - xz + cy \\ \dot{z} &= xy - bz + u_1 \end{aligned} \tag{24}$$

In order to convert Chen system equation to the general state of (1), the change of variable $r = y - x$ should be carried out. Therefore (24) would be converted to (25).

$$\begin{aligned} \dot{x} &= (2c - a)y - cr - (y - r)z \\ \dot{y} &= (2c - a)y - (c - a)r - (y - r)z \\ \dot{z} &= (y - r)y - bz + u_1 \end{aligned} \tag{25}$$

A. Stabilization of the States

In order to use the theorem, it is sufficient to establish (26) and (27).

$$\Phi_1(r, y) = 2c - a \tag{26}$$

$$\Phi_2(r, y) = c - a + \frac{2cy}{y - r} \tag{27}$$

According to the theorem, control signal and Lyapunov function will be obtained from (28) and (29).

$$\begin{aligned} u_1 = {} & \frac{\partial \Phi_2}{\partial r}\dot{r} + \frac{\partial \Phi_2}{\partial y}\dot{y} + y^2 - r^2 - k_1(z - \Phi_1) \\ & - k_2(z - \Phi_2) - (y - r)y + bz \end{aligned} \tag{28}$$

$$V(r, y, z) = \frac{1}{2}r^2 + \frac{1}{2}y^2 + \frac{1}{2}(z - \Phi_1)^2 + \frac{1}{2}(z - \Phi_2)^2 \tag{29}$$

According to the controller (28), (25) has been stabilized at the point $(0,0,\beta)$. In order to control Chen system to the origin point $(0,0,0)$, add a control input u_2 to (23). Thus the controlled system becomes:

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= (c-a)x - xz + cy + u_2 \\ \dot{z} &= xy - bz \end{aligned} \tag{30}$$

In order to use the theorem, it is sufficient to establish (31) and (32).

$$\Phi_1(x,z) = 0 \tag{31}$$

$$\Phi_2(x,z) = 0 \tag{32}$$

According to the theorem, the control signal will be obtained from (33).

$$u_2 = -cx - (k_1 + k_2 + c)y \tag{33}$$

$$V(x,y,z) = \frac{1}{2}x^2 + \frac{1}{2}z^2 + \frac{1}{2}(y-\Phi_1)^2 + \frac{1}{2}(y-\Phi_2)^2 \tag{34}$$

B. *Tracking Any Desired Trajectory*

In this section, we will find a control law u_2 so that a scaler output $x(t)$ of Chen system can track any desired trajectory $r(t)$. Let $\bar{x}$ be the deviation between the output x and the desired trajectory $r(t)$ i.e. $\bar{x} = x - r(t)$. Therefore, (26) would be converted to (35).

$$\begin{aligned} \dot{\bar{x}} &= a(y - \bar{x} - r) - \dot{r} \\ \dot{y} &= (c-a)(\bar{x}+r) - (\bar{x}+r)z + cy + u_2 \\ \dot{z} &= (\bar{x}+r)y - bz \end{aligned} \tag{35}$$

In order to use the theorem, it is sufficient to establish (36) and (37).

$$\Phi_1(\bar{x},z) = r + \frac{\dot{r}}{a} \tag{36}$$

$$\Phi_2(\bar{x},z) = 0 \tag{37}$$

According to the theorem, control signal and Lyapunov function will be obtained from (38) and (39).

$$\begin{aligned} u_2 &= -a\bar{x} - k_1(y-\Phi_1) - k_2(y-\Phi_2) \\ &\quad -(c-a)(\bar{x}+r) - cy \end{aligned} \tag{38}$$

$$V(\bar{x},y,z) = \frac{1}{2}\bar{x}^2 + \frac{1}{2}z^2 + \frac{1}{2}(y-\Phi_1)^2 + \frac{1}{2}(y-\Phi_2)^2 \tag{39}$$

V. Numerical Simulations

GBM is used as an approach to control Chaos in Chen system and eventually the results would be compared with Backstepping method [11] which is based on a recursive application of Lyapunov theory. Fig. 2, 3 and 4 show x, y and z of system can be stabilized with the control law u_1 to the bounded point $(0,0,\beta)$.

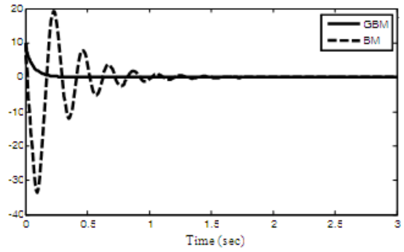

Fig. 2. Time response of the state *x*

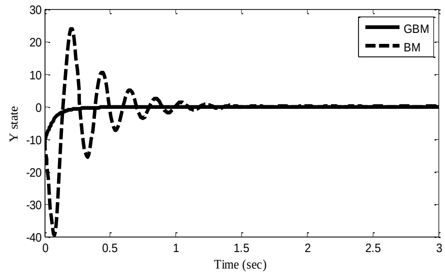

Fig. 3. Time response of the state *y*

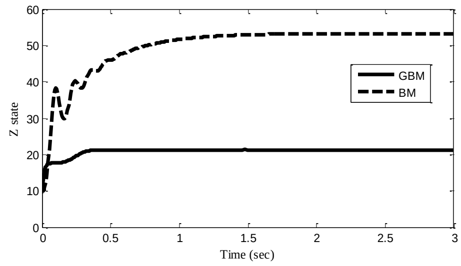

Fig. 4. Time response of the state *z*

Fig. 5, 6 and 7 show x, y and z of system can be stabilized with the control law u_2 to the bounded point $(0,0,0)$. Fig. 8 shows the control law u_2 (33) to the origin point $(0,0,0)$.

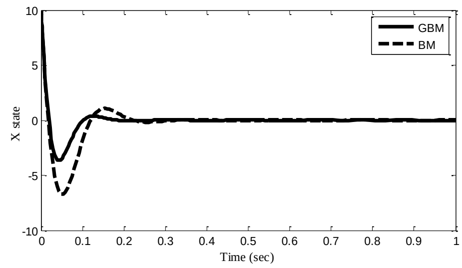

Fig. 5. Time response of the state *x*

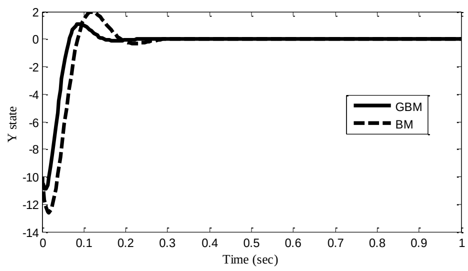

Fig. 6. Time response of the state *y*

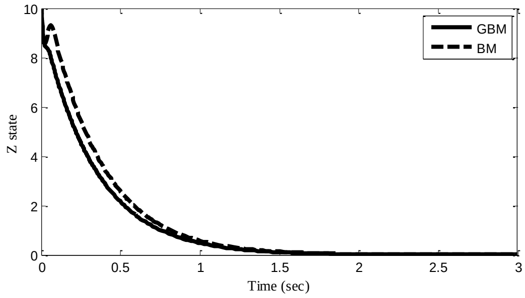

Fig. 7. Time response of the state *z*

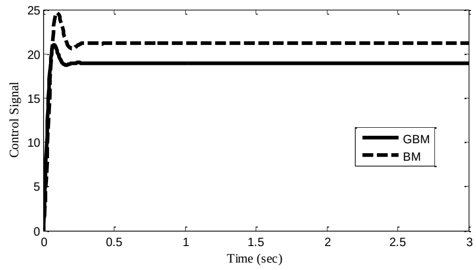

Fig. 8. Time response of the control input

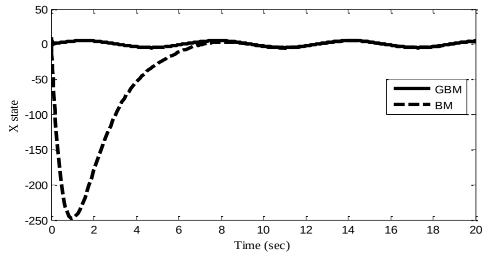

Fig. 9. Time response of *x* to track *r*(t)

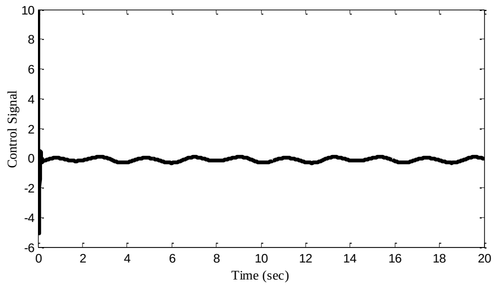

Fig. 10. Time response of the control input to track r(t)

Fig. 9 shows that x can track the desired trajectory $r(t) = 5\sin t$ with the control input u_2 is shown in Fig. 10.

VI. Discussion

By comparing the figures, the following results can be obtained.

- In GBM in relation to the BM [11], the system states are stabilized by a more limited control signal. Consequently, it is less possible that the control signal to be saturated.
- In the GBM in relation to the BM [11], control will be accomplished in a much shorter time and overshoot.

Considering the results obtained from simulations, the much more efficiency of GBM in relation to the BM will be demonstrated.

VII. Conclusion

In this study ¸a new method to control nonlinear systems is presented. The proposed method which is called Generalized Backstepping Method, by feed backing the dynamics of system and without eliminating the nonlinear dynamics, a controller is designed. A theorem is expressed for this method and the proof is given. Consequently, using this method, a controller is designed for the Chen chaotic system which is compared with the results obtained from the controller using the Backstepping Method.

The designed controller consists of parameters which acceptpositive values. The controlled system presents different behavior for different values. Improper selection of the parameters causes an improper behavior which may cause serious problems such as instability of system.This paper introduce adaptive neuro fuzzy control method which trained by different error data to achieve optimal parameters. By this approach the setting time and overshoot reach to their minimum values that demonstrated to have more optimal values when compared with previous methods.

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