Robust Control of Inverted Pendulum Using Fuzzy Sliding Mode Control and Genetic Algorithm

Mohamad Reza Dastranj, Mahbubeh Moghaddas, Younes Ghezi, and Modjtaba Rouhani

***Abstract*—one of the most important problems today is robotics and its control, due to the vast Application of inverted pendulum in robots. In this paper, by using a acombination of Fuzzy Sliding Mode methods and Genetic Algorithms, we have tired to optimally control the inverted pendulum by nonlinear equations. The results of this simulation have been mentioned in the conclusion. It seems that the results be acceptable results.**

***Index Terms*—Nonlinear, optimal, fuzzy sliding mode, genetic algorithm.**

I. Introduction

There are variety methods for Inverted Pendulum control that are presented since now. The presented methods for Inverted Pendulum control are divided generally in three groups. Classic methods such as PID PI controllers. [1], [2].Modern methods (adaptation-optimum,) [3], [4], [5].Artificial methods such as neural networks and fuzzy and Genetic Algorithm and PSO [6]-[8].theory are the presented methods for Inverted Pendulum angle control.

The design method in linear control comprise based on main application the wide span ' of frequency, linear controller has a weak application, because it can't compensate the nonlinear system effect completely.

II. Modeling an Inverted Pendulum

The cart with an inverted pendulum, shown below, is "bumped" with an impulse force, F. Determine the dynamic equations of motion for the system, and linearize about the pendulum's angle, theta = 0 (in other words, assume that pendulum does not move more than a few degrees away from the vertical, chosen to be at an angle of 0). Find a controller to satisfy all of the design requirements given below.

For this example, let's assume that

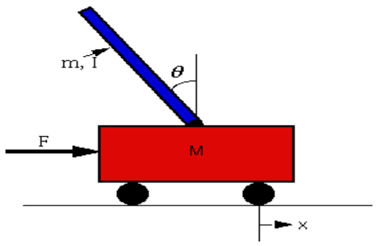

Fig. 1. The structure of an inverted pendulum

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TABLE I: Physical parameters of Inverted Pendulum

M	mass of the cart	0.5 kg
m	mass of the pendulum	0.2 kg
b	friction of the cart	0.1 N/m/sec
l	length to pendulum center of mass	0.3 m
I	inertia of the pendulum	0.006 kg*m^2
F	force applied to the cart	
x	cart position coordinate	
theta	pendulum angle from vertical	

This system is tricky to model in Simulink because of the physical constraint (the pin joint) between the cart and pendulum which reduces the degrees of freedom in the system. Both the cart and the pendulum have one degree of freedom (X and theta, respectively). We will then model Newton's equation for these two degrees of freedom.

$$\frac{d^2x}{dt^2} = \frac{1}{M}\sum_{cart} F_x = \frac{1}{M}\left(F - N - b\frac{dx}{dt}\right) \tag{1}$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{I}\sum_{pend} \tau = \frac{1}{I}(NL\cos(\theta) + PL\sin(\theta)) \tag{2}$$

It is necessary, however, to include the interaction forces N and P between the cart and the pendulum in order to model the dynamics. The inclusion of these forces requires modeling the x and y dynamics of the pendulum in addition to its theta dynamics. Generally, we would like to exploit the modeling power of Simulink and let the simulation take care of the algebra. Therefore, we will model the additional x and y equations for the pendulum.

$$m\frac{d^2x_p}{dt^2} = \sum_{pend} F_x = N \tag{3}$$

$$\Longrightarrow N = m\frac{d^2x_p}{dt^2} \tag{4}$$

$$m\frac{d^2y_p}{dt^2} = P - mg \tag{5}$$

$$\Longrightarrow P = m\left(\frac{d^2y_p}{dt^2} + g\right) \tag{6}$$

However, xp and yp are exact functions of theta. Therefore, we can represent their derivatives in terms of the derivatives of theta.

$$x_p = x - L\sin(\theta) \tag{7}$$

$$\frac{dx_p}{dt} = \frac{dx}{dt} - L\cos(\theta)\frac{d^2\theta}{dt^2} \tag{8}$$

$$\frac{d^2x_p}{dt^2}=\frac{d^2x}{dt^2}+L\,sin(\theta)\left(\frac{d\theta}{dt}\right)-L\,cos(\theta)\frac{d^2\theta}{dt^2} \tag{9}$$

$$y_p=L\,cos(\theta) \tag{10}$$

$$\frac{dy_p}{dt}=-L\,sin(\theta)\frac{d\theta}{dt} \tag{11}$$

$$\frac{d^2y_p}{dt^2}=-L\,cos(\theta)\left(\frac{d\theta}{dt}\right)^2-L\,sin(\theta)\frac{d^2\theta}{dt^2} \tag{12}$$

These expressions can then be substituted into the expressions for N and P. Rather than continuing with algebra here, we will simply represent these equations in Simulink.

Simulink can work directly with nonlinear equations.

III. Sliding Mode Controller

Nonlinear system control that its model isn't clear carefully works with tow methods:

(1)Robust control methods.

(2) Adaptive control methods.

In control view, uncertainly in modeling is divided in two main kinds:

(1) Non certainly in existent Para meters in model

(2) Estimating the lower step for system and being UN modeled dynamics in the estimating model.

Sliding control is one of the designed modes for robust control that make access to system desired application estimating system in model.

The major idea of this method is the controlling of nonlinear first grade system is easier than n grade system control in spite of uncertainly.

But this function maybe cause the control law with more energy that is not practicable implement tat ion.

Sliding mode is really compromise between modeling and suitable operation with inaccurate design.

We consider the non linear system model in this rule:

$$X^n=f(x)+b(x)u \tag{13}$$

That *F(x)* is nonlinear function, its high boundary characterized as X function.

B(x) is a continuous function that its high and low boundaries characterized by X function.

The good of finding X is in this way that in *g(x) F(x)* function we can follow the desirable mode in spite of uncertainly.

$$\widetilde{X}=X-X_d=[\tilde{X},\tilde{X}',\ldots,\tilde{X}^{n-1}] \tag{14}$$

In ideal state

$$\tilde{X}=0 \tag{15}$$

Sliding surface equation defines as below:

$$s=e'+\alpha_1 e+\alpha_2\int e\,dt \tag{16}$$

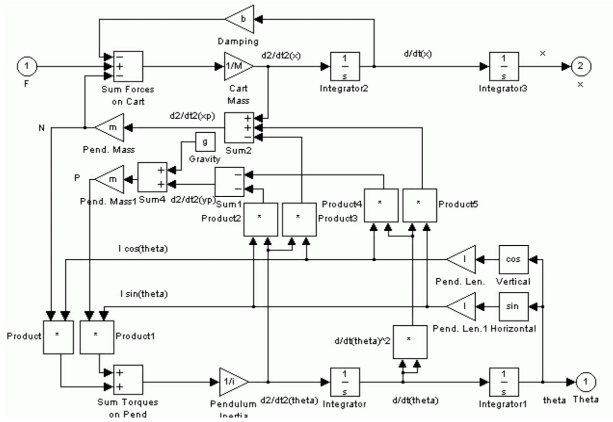

Fig. 2. The block diagram of an inverted pendulum

Because of the signals of control that gain with this designing method has limited energy, it is necessary to:

$$X_d(0)=X(0) \tag{17}$$

in other word:

$$S(X,t)\equiv 0 \tag{18}$$

$$\frac{1}{2}\frac{dS^2}{dt}\le -\eta|S| \tag{19}$$

In designing, the control low on S(t) continuously is noticed cause we should concentrate to carelessness in model in sliding surface and reduced the chattering effect.

We can write the system's dynamics when in some situation they are in sliding state.

$$S'=0 \tag{20}$$

The gained control signals for this system are as below:

$$U=k_1\times out_{fuzzy}*S \tag{21}$$

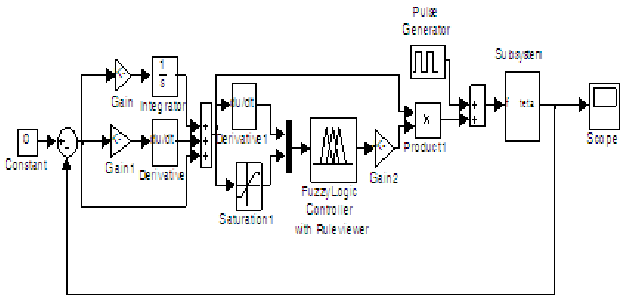

Fig. 3.simulink block diagram of FSMC

Fuzzy controls are designed based on created sliding surface and sliding surface changes. All of the fuzzy rules collection came in Table II

TABLE I: Fuzzy Rule

dS / S	**NB**	**NS**	**ZE**	**PS**	**PB**
N	**B**	**B**	**M**	**S**	**B**
Z	**B**	**M**	**S**	**M**	**B**
P	**B**	**S**	**M**	**B**	**B**

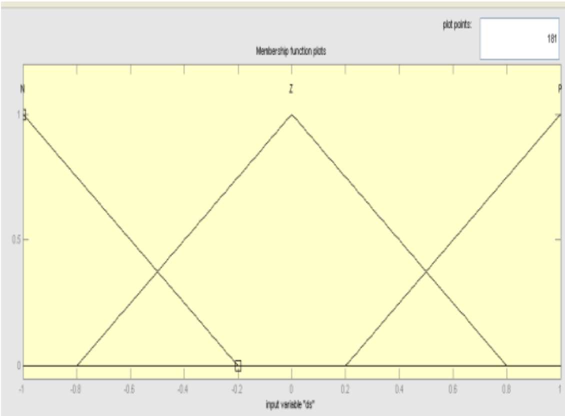

Fig. 4. Membership functions for (ds/dt) normalized inputs

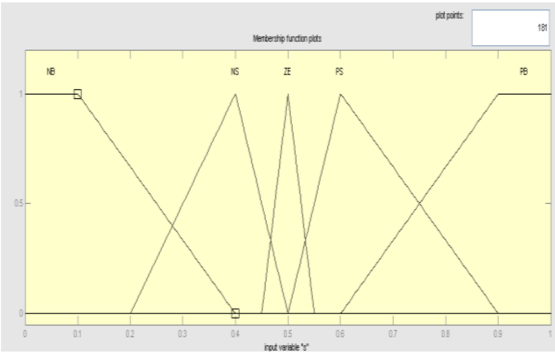

Fig. 5. Membership functions for (s) normalized inputs

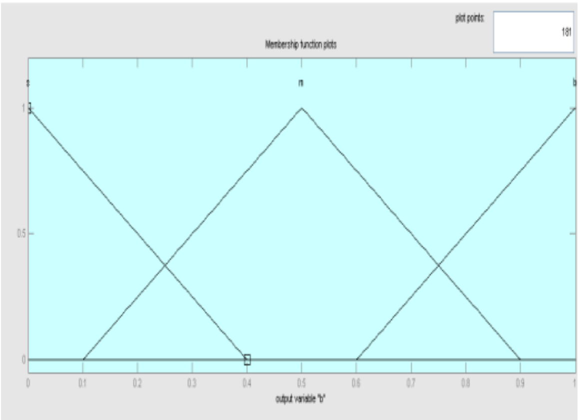

Fig. 6. Membership functions for (out fuzzy) normalized outputs

IV. Genetic Algorithm

In this algorithm, first of all, we create some random populations. Every individual (gene) In GA is considered in the form of binary strings then, fitness for every individual is chosen with regard to its fitness.

For creating the next generation, three stage is the selection phase, which consists of different phases, including ranking, proportional and.. .and the second phase is the combination phase. In this phase, the two parents are combined with pc possibility and the next generation comes in to being. By considering that during the past phases of gene it may cause noise, in fact, this phase is a Random noise which causes a small pc possibility for every bit.

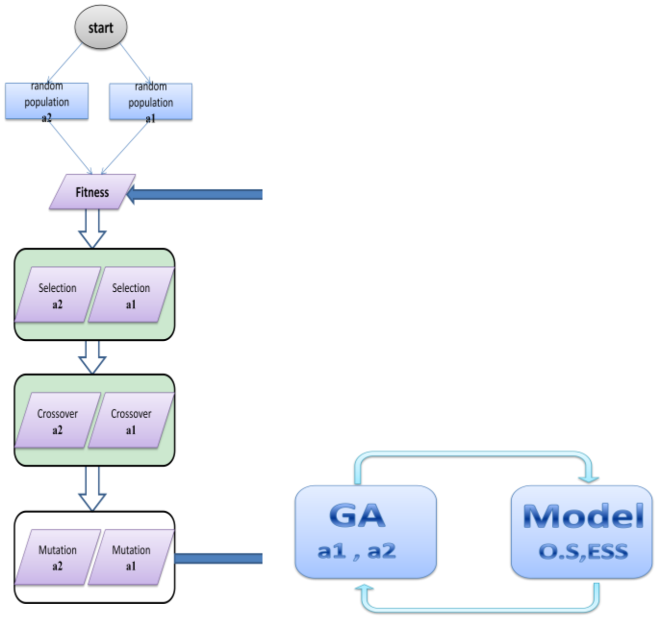

Fig. 7. structure of genetic algorithm

A. Fitness Function

For GA in every problem, a fitness function must be defined. F functions can be described as follows:

$$F = OverShoot + Ess \quad (22)$$

$$F = A * OverShoot + B * Ess \quad (23)$$

$$F = e^{A*OverShoot + B*Ess} \quad (24)$$

In this problem, the aim is to minimize every function of F.As GA Has the ability to be maximized, hence, fitness function is defined as below.

$$Fitnes = K - F \quad (25)$$

$$Fitnes = \frac{1}{F} \quad (26)$$

If the fitness function is selected from an equation (25)constant parameter k must be regulated in a way That causes no harm to the problem. If k is a small number, fitness will be negative and for the capital k, the fitness of all the individuals in the society will be approximated .In this paper, some equations have been used.

V. Conclusion

In this paper, a robust control system with the fuzzy sliding mode controller and the additional compensator is presented for a Inverted Pendulum position control. According to the simulation results, the FSMC controllers can provide the properties of insensitivity and robustness to uncertainties and external disturbances, and response of the Inverted Pendulum for FSMC controllers against uncertainties and external disturbance is the same Fuzzy sliding mode controller gives a better response to system than the fuzzy and classical PID controllers if α_1, α_2, k_1 control parameters set suitably.

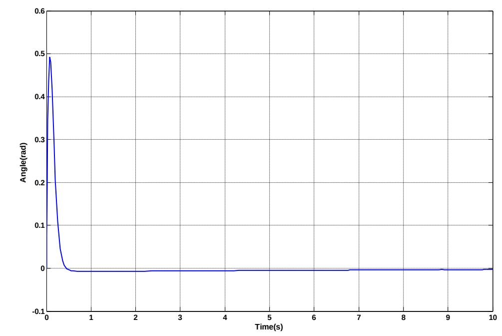

Fig. 8. Inverted pendulum rod angle for initial 0.1 radians(best result

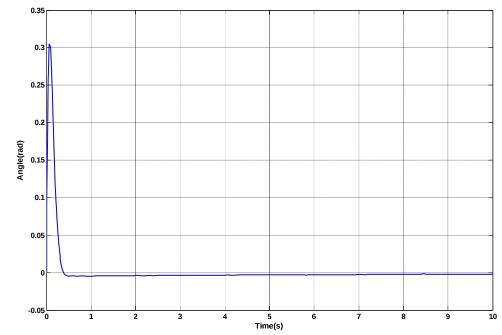

Fig. 9. Inverted pendulum rod angle for initial 0.3 radians(best result)

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