Modified Root MUSIC for Adjacent Coherent Sources

Hassan Mohamed El-Kamchouchi and Ahmed Samir El-Torgoman

***Abstract*—Root MUSIC is one of the most important algorithms that address the DoA issue for its discrete estimations for non-coherent sources. However, coherent signals cannot be discriminated by using the algorithm, even they are adjacent in this paper. We will concentrate on utilizing root MUSIC advantages discrete estimation for DoA by using as simple modification for covariance matrix in root MUSIC algorithm to capable to handle signals of coherent sources as well as factors related to number of snapshots and number of element array. As well will result in enhancing ROOT MUSIC algorithm resolution for adjacent coherent sources to 1 degree space which will be called ECRMUSIC (Enhanced Coherent-Root MUSIC).**

***Index Terms*—Coherent sources, direction of arrival, resolution enhancement, root MUSIC.**

I. Introduction

Smart antennas combine multiple antenna elements with signal processing capability to optimize radiation / reception [1], [2] is divided to two main sections beam forming signal is formed due to signal coming from target which decrease power consumption and nulls interferers jamming signal and frequency reuse will be used within cell space called space division multiple access SDMA [3]. The second section of smart antenna is direction of arrival which is so important for finding the direction from which the signal is coming from many algorithms are studied in this area [1], [4]-[6]. It is important in many applications like sonar / rescue devices wireless communication, algorithms addressing this technique divided into three main categories conventional & beam forming and subspace techniques as [1]. MUSIC / ESPIRIT / BARTLETT / CAPON, the implementation is used to eliminate interference & combine the required signals to improve performance, from literature the common problems for this algorithm 2 adjacent signals cannot be. In this paper, we will address the problem of coherent sources with MUSIC algorithm by inserting transition matrix to correct covariance matrix de efficient a simpler way than spatial smoothing method as well as enhancing resulting resolution by inserting enhancement factors [2]. As no of snapshots, no of elements, which will result in enhancing root MUSIC algorithm resolution for coherent sources to 1 degree space which will be called EC-Root MUSIC (Enhanced coherent-Root MUSIC). The paper is organized as follow: presentation of basic MUSIC algorithm, coherent sources and its problem of detection, how it is solved is in Section II. Presentation of enhancing factors is in Section III, seeing the results in Section IV. Section V is conclusion and future work. A road map of procedure is presented in Fig. 1.

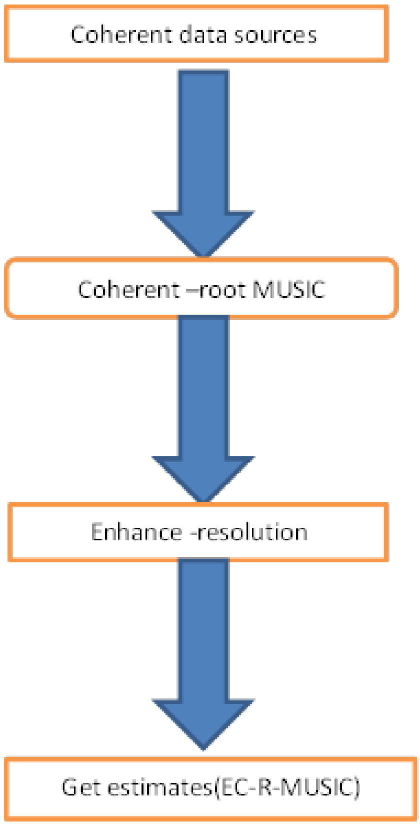

Fig. 1. Road map of procedure.

II. Coherent Root MUSIC Overview

Coherent Root MUSIC based on MUSIC algorithm [7] in which transition matrix is conjugate re-resolution construction of the data matrix of the MUSIC algorithm which root MUSIC based on will be presented first at its normal case.

A. Root MUSIC Algorithm

This algorithm had been introduced as an improvement of MUSIC algorithm [8] by Barabell [9]. Using the MUSIC spectrum function, the root of the polynomial is used to estimate the angles of arrival. This algorithm is more practical, since the results are given in a numerical format instead of spectrum plotting in MUSIC algorithm. The algorithm can be summarized by the following steps:

$$\text{PMUSIC}(\phi)=\frac{1}{S^H(\phi)Q_nQ_n^HS(\phi)} \tag{1}$$

where (ϕ) is DoA and S and Q are the signal and noise subspace respectively and superscript H is Hermitian operator Note that since the eigenvectors making up Qn are

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orthogonal to the signal steering vectors, the denominator becomes zero when ϕ is a signal direction. (1) can be simplified as

$$c = Q_n Q_n^H \quad (2)$$

$$P_{music}(\phi) = \frac{1}{|s(\phi)^H cs(\phi)|} \quad (3)$$

For a uniform linear array the mth element of the array steering vector is written as

$$a_m(\phi) = e^{jkd(m-1)sin\phi} \quad m = 1,2,\ldots,M \quad (4)$$

The denominator can be written as

$$s(\phi)^H cs(\phi) = \sum_{m=1}^{M}\sum_{n=1}^{N} e^{-jkd(m-1)sin\phi}\, C_{mn} e^{e^{jkd(n-1)sin\phi}}$$
$$= \sum_{l=-M+1}^{M+1} c_l\, e^{jkdlsin\phi} \quad (5)$$

where c_l is the sum of the diagonal element of C along the l^{th} diagonal such that

$$C_l = \sum_{n-m=1} C_{mn} \quad (6)$$

Equation (6) can be simplified in polynomial form whose coefficients C_l.

$$D(z) = \sum_{l=-M+1}^{M-1} C_l z^l \quad (7)$$

where z=$e^{-jkdsin\phi}$. The roots of $D(z)$ that lie closest to the unit circle correspond to the poles of the MUSIC pseudospectrum. Thus, this technique is called Root MUSIC. These roots which has magnitude = 1 i.e. at unit circle is then employed into

$$\phi_i = \sin^{-1}\left(\frac{1}{kd}\arg(z_i)\right) \quad (8)$$

To get the direction of arrival of impinging signals.

B. Coherent Sources Overview

The problem involving coherent sources is a fatal problem for subspace algorithms [10]. When there is a coherent signal in the signal source, the signal covariance matrix is rank defective. In this case, the original super-resolution algorithm will fail. Therefore, it will greatly affect the performance of DOA estimation (Fig. 2).

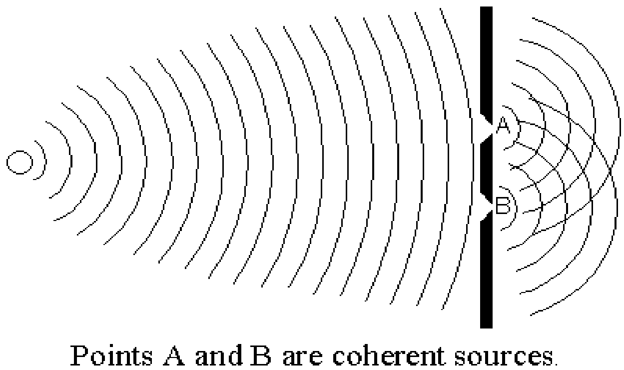

Fig. 2. Coherent Sources illustration [11].

C. Coherent Sources Problems in Root MUSIC

Highly correlated or coherent source signals are common in multipath propagation environments due to the reflection and refraction of source signals in practical. Based on such scenario, the coherent sources facilitate the rank loss of the covariance matrix, which could result in the failure of the conventional high-resolution estimation algorithms.

1) How it is solved?

Covariance matrix rank loss is addressed by many methods to overcome its bad effect in direction of arrival estimation. A simple subspace method is used to construct polynomial function and obtain the roots then estimated DOA directly found, which is proposed by [12] conjugate reconstruction of covariance matrix of MUSIC algorithm.

2) Methodology

Make a transformation matrix *J*, *J* is an Mth-order anti-matrix, known as the transition matrix, i.e.

$$\begin{bmatrix} \mathbf{0} & \cdots & \mathbf{1} \\ \vdots & \ddots & \vdots \\ \mathbf{1} & \cdots & \mathbf{0} \end{bmatrix}$$

Let *Y=JX**, where *X** is the complex conjugate of *X*, then the covariance of data matrix *Y* is

$$R_y = E[YY^H] = JRX^*J. \quad (9)$$

From the sum of R_x and R_y, the reconstructed conjugate matrix can be obtained.

$$R = Rx + Ry = AR_sA^H + J[AR_sA^H]^*J + 2\sigma^2 I \quad (10)$$

According to matrix theory, the matrices R_x, R_y and R have the same noise subspace. To conduct characteristic decomposition of R and get its eigenvalue and eigenvector, according to the estimated number of signal source, separate the noise subspace, and then use this new noise.

III. Enhancing Resolution

The last phase in this method to enhance resolution of very close angles of DoA When the separation angles between sources are very small, rootMUSIC algorithm degrade estimate the angles correctly. Thus, an improvement for that algorithm was proposed by adding new factors to achieve this goal and estimate the adjacent angles correctly. This was achieved even with separation of degree only between sources. These factors are related to signal and antenna parameters, which are affecting the accuracy of estimation algorithm. Those factors represent the ratio by which each corresponding parameter should be increased in order to get the required super high resolution. Those factors will virtually maximize the values of the corresponding parameters during calculation and enhance the estimation accuracy of the algorithms. Those factors are defined as follows as:

A. Number of Snapshots Factor (SF)

This factor represents the ratio for incrementing the snapshots. The multiplication of *SF* with the snapshot number during processing will give higher number of snapshots. Thus, increasing the efficiency of the estimation and obtain sharper estimates in root MUSIC. By adding more snapshots, the estimation variance for covariance matrix will decrease and thus sharper estimates for root MUSIC are obtained. However, very high values for *SF* will lead to processing delay since the processing time will also increase. Hence, suitable value for it should be selected according to the requirements. The new number of snapshot *N'* can then be written as

$$N' = N \times SF \tag{11}$$

B. Number of Elements Factor (EF)

EF represents the ratio for incrementing the number of elements in the array *M*. It had been added to raise the value of number of elements during processing. This was done by multiplying *EF* with the actual number of elements in the array. Thus, it gives more signal component for the algorithm. Here the importance of this modification appears during manufacturing process. By selecting the suitable *EF* value, the suitable number of elements required for the antenna design can be selected easily. Then we can substitute the new number of element M' in. M' can be expressed by:

$$M' = M \times EF \tag{12}$$

IV. Simulation Steps and Results

Simulation done with MATLAB ®2015a release to clarify obtained results we have to see it compared with results obtained without any improvement with the raw-Root MUSIC algorithm as a main frame. We will begin with incoherent sources and Raw-Root MUSIC algorithm with 2 direction of arrival incidence (30°, 70°) degrees as in Table I:

TABLE I: Incoherent Sources and Raw Root MUSIC

Direction	Estimate	Error%
30°	30.00°	0%
70°	69.98°	0.02%

Then coherent sources with the same frequency and same incidence angles as last case and raw MUSIC algorithm we find performance degradation as in Table II, which any way is better than MUSIC Algorithm but still coherent sources limit the Raw-Root MUSIC algorithm performance.

TABLE II: Coherent Raw with Basic Root MUSIC

Direction	Estimate	Error%
30°	28.33°	5.56%
70°	72.03°	-2.9%

Then check again with coherent root MUSIC algorithm the roots appear only at the estimated angles with high accuracy as seen in Table III.

TABLE III: Coherent Sources with Coherent Root MUSIC

Direction	Estimate	Error%
30°	30.07°	-0.23%
70°	69.90°	0.14%

But when incident angles is adjacent (30°, 31°) the coherent MUSIC algorithm failed to discriminate them as in Table IV.

TABLE IV: Adjacent Coherent Sources with Coherent Root MUSIC without Enhancing Resolution Factors

Direction	Estimate	Error%
30°	30.41°	-1.36%
31°	30.60°	1.29%

But when we use Enhanced coherent MUSIC with enhancing resolution factors (*EF*) and (*SF*) with limited ratio to see its impact on results we find incredible accuracy as we see in the Table V and Table VI for snapshots mainly =200 and number of element array = 10, we get the following results.

TABLE V: Adjacent Coherent Signal Estimation with EC-Root MUSIC *EF*=1.5, *SF*=3

Direction	Estimate	Error%
30°	29.98°	0.067%
31°	30.96°	0.13%

TABLE VI: Adjacent Coherent Signal Estimation with EC-Root MUSIC *EF*=3, *SF*=5

Direction	Estimate	Error%
30°	30.0023°	$3*10^{-3}$%
31°	31.0023°	$3*10^{-3}$%

TABLE VII: Adjacent Coherent Signal Estimation with EC-Root MUSIC *EF*=5, *SF*=25

Direction	Estimate	Error%
30°	29.99°	0.033%
31°	30.99°	0.032%

Table VII shows that increasing enhancing resolution factors has threshold as the application used may demand for increasing factors without threshold will result in processing delay and poor performance but any way better than the raw Root MUSIC case.

V. Conclusion and Future Work

In this paper, the theory of smart antennas and the basic concept of Root MUSIC DOA algorithms had been illustrated and discussed. That algorithm when implemented in smart antenna will lead to efficient use of networks. To achieve this goal, the estimation of source direction should be accurate even if sources are too close to each other in angles and coherent, which is achieved through covariance matrix correction and adding enhancing factors that led at last to accurate estimation for direction of arrival for adjacent coherent signals. However, excessive increment for factors values will lead to degradation in performance. So that, suitable values for factors should be selected.

Enhancing resolution of DoA for coherent signals is now an important demand in today's world of smart antennas, this work can be modified to update estimation in uniform circular arrays as well as Mixed signals identification in Multipath environment will be promising land for further research.

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