# Transverse Electromagnetic Modes of Menger Sponge

M. J. Mughal, M. Omar, Safiullah Khan, and A. Noor

***Abstract*—In this article, an analytical solution for the electromagnetic modes of a Menger sponge using field equations of fractional space is presented. In order to realize the electromagnetic modes in non-integer dimension space, the fields inside the Menger sponge are expressed using a fractional parameter $2 \leq D \leq 3$. The modes are calculated under two conditions of boundaries i.e., PMC and PEC, for $D = 3$ correspond to ordinary integer dimension space. Generalized solution for lossless and source free medium is studied for non-integer dimension. As anticipated, the dimension of media have effect on electric and magnetic fields. The classical results are recovered when integer dimensional space is considered. The proposed solution is useful for deriving electromagnetic modes of fractal structures.**

***Index Terms*—Fractional dimension, menger spone, transverse electromagnetic.**

## I. Introduction

Many shapes in nature are complex and cannot be define by Euclidean geometry. Mandelbrot introduced the concept of "fractals" for complex structures [1]. An important property of fractals is that they are self-similar and repeats themselves at different scales. Hence they can be define with very less number of parameters. Many natural occurring structures and geometries like roughness of ocean floor, snow, earth and even galaxies are examples of fractional dimension. Fractional calculus is an important tool to study the behavior of homogenous models at fractal interfaces [2], [3]. This study provides a way to introduce solutions of electromagnetic problems in fractional dimension space [4], [5]. In 1996, fractional integration was used to find solution for the scaler wave equation and then later source distributions which are equivalent to fractional dimension Dirac delta function, were analysed [6], [7]. Therefore, it is worthwhile to generalize the theories of electromagnetics in order to get full benefits of these highly complex structures. In this respect from last few decades they are the subject of interest for many researchers [8]-[14].

Menger sponge is a good example of fractal structure,

described first time by Karl Menger [15]. Menger sponge is a symmetric and self-similar fractal cube. Menger sponge does not occupy integer-dimension because of its infinite surface area and zero volume, it can only be defined in fractional dimensions.

There are not many analytical solution of the wave equation for fractal structures available in literature todate, hence the wave equation is solved using fractional space formulation. Previously such structures were characterized using numerical and experimental methods only. However, using fractional space formulation it is now possible to obtain the analytical results of fractal structures [16], [17].

In this paper, transverse magnetic (TM) and transverse electric (TE) eigenmodes of Menger sponge are derived analytically using fractional space formulation. This method can also be extend to other self-similar fractals like Sierpinski carpet. Section II covers TM modes. TE modes which occur due to the boundary conditions, are discussed in Section III. In Section IV, it is shown that classical results can be recovered from fractional space, when integer dimensions are inserted. This approach permits to obtain results regarding behavior of complex fractal structures.

## II. Transverse Electromagnetic Modes of Menger Sponge

The general solutions of fractional dimension case are valid only for large arguments. Hence Menger sponge placed at ($x_0$, $y_0$, $z_0$) far from the origin (0, 0, 0) such that $\beta_{x0}$, $\beta_{y0}$, $\beta_{z0}$ $\gg 1$. Let $a$, $b$ and $c$ are the width, height and length and of Menger sponge respectively and $\eta \gg \eta_0$, where $\eta_0$ is intrinsic impedance of free space and $\eta$ is intrinsic impedance of Menger sponge. The boundaries of Menger sponge are approximated as perfect magnetic conductor (PMC), shown in Fig. 1. The wave equations for source free and lossless media that describe complex electric and magnetic field intensities are given by Helmholtz's equations [18], as follows:

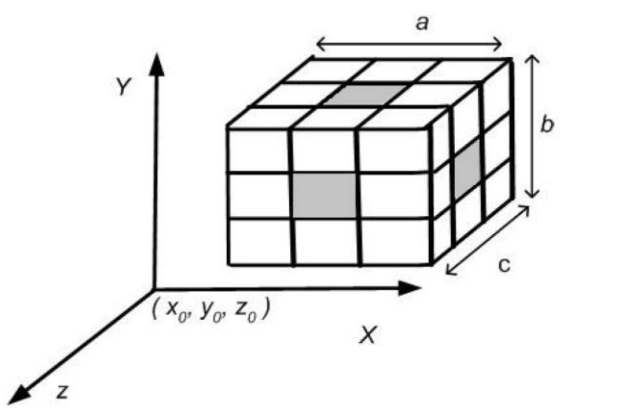


Fig. 1. Mengersponge placed far from the origin.

$$\begin{aligned} \nabla_D^2 E + \beta^2 E &= 0 \\ \nabla_D^2 H + \beta^2 H &= 0 \end{aligned} \tag{1}$$

where wave number, $\beta^2 = \omega 2 \mu \epsilon$ , $\nabla^2{}_D$ is the scalar Laplacian operator in $D$-dimensional fractional space [19]. For

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simplicity electric field has only $x$-component i.e., $E_y = E_z = 0$. The general solution of electric fields for fractional space [16], which corresponds to a $TM^x$ mode is,

$$\begin{aligned} E(x,y,z) &= E_x(x,y,z) \\ &= x^{n_1} y^{n_2} z^{n_3} \left[ C_1 J_{n_1}(\beta_x x) + C_2 Y_{n_1}(\beta_x x) \right] \\ &\left[ C_3 J_{n_2}(\beta_y y) + C_4 Y_{n_2}(\beta_y y) \right] \left[ C_5 J_{n_3}(\beta_z z) + C_6 Y_{n_3}(\beta_z z) \right] \end{aligned} \tag{2}$$

where $n_1 = 1 - \alpha_1/2$, $n_2 = 1 - \alpha_2/2$ and $n_3 = 1 - \alpha_3/2$, $J_{n1}(\beta_x x)$ and $Y_{n1}(\beta_x x)$ are Bessel functions of first and second kinds of order $n_1$, where $\alpha_1$, $\alpha_2$ and $\alpha_3$ are parameters used to describe measure distribution of space. Dimension of the system can be described as $D = \alpha_1 + \alpha_2 + \alpha_3$. For large arguments of Bessel functions, above equation can also be written as below [20],

$$\begin{aligned} &E_x(x,y,z) = x^{n_1 - \frac{1}{2}} y^{n_2 - \frac{1}{2}} z^{n_3 - \frac{1}{2}} \times \\ &\left[ D_1 \cos\left( \beta_x x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) + D_2 \sin\left( \beta_x x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) \right] \\ &\left[ D_3 \cos\left( \beta_y y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) + D_4 \sin\left( \beta_y y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) \right] \\ &\left[ D_5 \cos\left( \beta_z z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) + D_6 \sin\left( \beta_z z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) \right] \end{aligned} \tag{3}$$

for simplicity,

$$C_i \sqrt{\frac{2}{\pi \beta_x x}} = D_i \qquad (i = 1, 2, \ldots, 6) \tag{4}$$

corresponding magnetic field in fractional space can be determine using Maxwell equation modified for fractional space [21],

$$curl_{\mathfrak{D}} E(x,y,z) = -j\omega\epsilon H(x,y,z) \tag{5}$$

where [20],

$$\begin{aligned} curl_{\mathfrak{D}} E(x,y,z) &= \left[ \frac{\partial}{\partial z} E_x + \frac{1}{2} \frac{\alpha_3 - 1}{z} E_x \right] \hat{y} \\ &- \left[ \frac{\partial}{\partial y} E_x + \frac{1}{2} \frac{\alpha_2 - 1}{y} E_x \right] \hat{z} \end{aligned} \tag{6}$$

equation of magnetic field in fractional space is as follows:

$$\begin{aligned} &H(x,y,z) = \frac{j}{\mu\omega} x^{n_1 - \frac{1}{2}} y^{n_2 - \frac{1}{2}} z^{n_3 - \frac{1}{2}} \times \\ &\left\{ \left[ D_1 \cos\left( \beta_x x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) + D_2 \sin\left( \beta_x x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) \right] \right. \\ &\left[ D_3 \cos\left( \beta_y y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) + D_4 \sin\left( \beta_y y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) \right] \\ &\left[ -\beta_z D_5 \sin\left( \beta_z z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) + \beta_z D_6 \cos\left( \beta_z z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) \right] \hat{y} \\ &- \left[ D_1 \cos\left( \beta_x x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) + D_2 \sin\left( \beta_x x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) \right] \\ &\left[ -\beta_y D_3 \sin\left( \beta_y y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) + \beta_y D_4 \cos\left( \beta_y y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) \right] \\ &\left. \left[ D_5 \cos\left( \beta_z z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) + D_6 \sin\left( \beta_z z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) \right] \hat{z} \right\} \end{aligned} \tag{7}$$

Boundary conditions are $\mathbf{H}_y = 0$ at $z = z_0$, $z = z_0 + c$, $x = x_0$ and $x = x_0 + a$. Using the condition $\mathbf{H}_y = 0$ at $x = x_0$, $x = x_0 + a$, $D_1 = 0$ and $D_2 \neq 0$,

$$\begin{aligned} x_0 &= \frac{N\pi + \frac{\pi}{4} + \frac{n_1 \pi}{2}}{\beta_x} \qquad (N = 1,2,3\ldots) \\ \beta_x &= \frac{L\pi}{a} \qquad (L = 1,2,3\ldots) \end{aligned} \tag{8}$$

Similarly, from the condition $\mathbf{H}_y = 0$ at $z = z_0$, $z = z_0 + c$, $D_6 = 0$ and $D_5 \neq 0$,

$$\beta_z = \frac{P\pi}{c} \qquad (P = 1,2,3\ldots) \tag{9}$$

for applying condition on $z$-component $\mathbf{H}_z = 0$ at $y = y_0$, $y = y_0 + b$, $D_4 = 0$ and $D_3 \neq 0$,

$$\beta_y = \frac{Q\pi}{b} \qquad (Q = 1,2,3\ldots) \tag{10}$$

The wave number for TM mode,

$$\beta = \sqrt{\left( \frac{L\pi}{a} \right)^2 + \left( \frac{Q\pi}{b} \right)^2 + \left( \frac{P\pi}{c} \right)^2} \tag{11}$$

and finally, the electric and magnetic field inside a Menger sponge are calculated as,

$$\begin{aligned} &E_x(x,y,z) = C x^{n_1 - \frac{1}{2}} y^{n_2 - \frac{1}{2}} z^{n_3 - \frac{1}{2}} \sin\left( \frac{L\pi}{a} x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) \\ &\cos\left( \frac{Q\pi}{b} y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) \cos\left( \frac{P\pi}{c} z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) \end{aligned} \tag{12}$$

$$\begin{aligned} &H(x,y,z) = \frac{j}{\mu\omega} C x^{n_1 - \frac{1}{2}} y^{n_2 - \frac{1}{2}} z^{n_3 - \frac{1}{2}} \times \\ &\left[ -\beta_z \sin\left( \frac{L\pi}{a} x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) \cos\left( \frac{Q\pi}{b} y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) \right. \\ &\left. \sin\left( \frac{P\pi}{c} z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) \right] \hat{y} + \left[ \beta_y \sin\left( \frac{L\pi}{a} x - \frac{\pi}{4} - \frac{n_1 \pi}{2} \right) \right. \\ &\left. \sin\left( \frac{Q\pi}{b} y - \frac{\pi}{4} - \frac{n_2 \pi}{2} \right) \cos\left( \frac{P\pi}{c} z - \frac{\pi}{4} - \frac{n_3 \pi}{2} \right) \right] \hat{z} \end{aligned} \tag{13}$$

where $C = D_2D_3D_5$ and (14) , (15) are the electric and magnetic field equations, respectively for PMC boundary. As Menger sponge is symmetric in pattern i.e., $a = b = c$, for Menger sponge D $\simeq$ 2.727, hence $n_1 = n_2 = n_3 = 0.5455$. The above fields equations can be approximated as,

$$E_x(x,y,z) = Cx^{0.0455}y^{0.0455}z^{0.0455} \times \sin\left(\frac{L\pi}{a}x - \frac{\pi}{4} - \frac{0.5455\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y - \frac{\pi}{4} - \frac{0.545\pi}{2}\right)\cos\left(\frac{P\pi}{c}z - \frac{\pi}{4} - \frac{0.545\pi}{2}\right) \tag{14}$$

classical results can be recovered [18], when the dimension is integer i.e., $D = 3$ and $\alpha_1 = \alpha_2 = \alpha_3 = 1$. The equation of electric field becomes,

$$E(x,y,z) = E_x(x,y,z) = C\sin\left(\frac{L\pi}{a}x - \frac{\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y - \frac{\pi}{2}\right)\cos\left(\frac{P\pi}{c}z - \frac{\pi}{2}\right) \tag{15}$$

## III. Transverse Electric Modes of Menger Sponge

Menger sponge placed at $(x_0, y_0, z_0)$ far from the origin (0, 0, 0) and $\eta_0 \gg \eta$. The boundaries of Menger sponge can now be approximated as perfect electric conductor (PEC). The general solution of (2), which corresponds to a $TE^x$ mode is,

$$H(x,y,z) = H_x(x,y,z) = x^{n_1}y^{n_2}z^{n_3}\left[F_1J_{n_1}(\beta_x x) + F_2Y_{n_1}(\beta_x x)\right]\left[F_3J_{n_2}(\beta_y y) + F_4Y_{n_2}(\beta_y y)\right]\left[F_5J_{n_3}(\beta_z z) + F_6Y_{n_3}(\beta_z z)\right] \tag{16}$$

for large arguments of Bessel functions, above equation can also be written as below,

$$H_x(x,y,z) = x^{n_1-\frac{1}{2}}y^{n_2-\frac{1}{2}}z^{n_3-\frac{1}{2}} \times \left[G_1\cos\left(\beta_x x - \frac{\pi}{4} - \frac{n_1\pi}{2}\right) + G_2\sin\left(\beta_x x - \frac{\pi}{4} - \frac{n_1\pi}{2}\right)\right]\left[G_3\cos\left(\beta_y y - \frac{\pi}{4} - \frac{n_2\pi}{2}\right) + G_4\sin\left(\beta_y y - \frac{\pi}{4} - \frac{n_2\pi}{2}\right)\right]\left[G_5\cos\left(\beta_z z - \frac{\pi}{4} - \frac{n_3\pi}{2}\right) + G_6\sin\left(\beta_z z - \frac{\pi}{4} - \frac{n_3\pi}{2}\right)\right] \tag{17}$$

for simplicity,

$$F_i\sqrt{\frac{2}{\pi\beta_x x}} = G_i \quad (i = 1,2,\ldots,6) \tag{18}$$

corresponding electrical field in fractional space can be determine using Maxwell equation [21],

$$\mathfrak{curl}_{\mathfrak{D}}H(x,y,z) = j\omega D \tag{19}$$

where,

$$\mathfrak{curl}_{\mathfrak{D}}H(x,y,z) = \left[\frac{\partial}{\partial z}H_x + \frac{1}{2}\frac{\alpha_3 - 1}{z}H_x\right]\hat{y} - \left[\frac{\partial}{\partial y}H_x + \frac{1}{2}\frac{\alpha_2 - 1}{y}H_x\right]\hat{z} \tag{20}$$

equation of electrical field in fractional space is as follows:

$$\begin{aligned}E(x,y,z) = -\frac{j}{\mu\epsilon}x^{n_1-\frac{1}{2}}y^{n_2-\frac{1}{2}}z^{n_3-\frac{1}{2}} \times \Bigg\{&\left[G_1\cos\left(\beta_x x - \frac{\pi}{4} - \frac{n_1\pi}{2}\right) + G_2\sin\left(\beta_x x - \frac{\pi}{4} - \frac{n_1\pi}{2}\right)\right]\\ &\left[G_3\cos\left(\beta_y y - \frac{\pi}{4} - \frac{n_2\pi}{2}\right) + G_4\sin\left(\beta_y y - \frac{\pi}{4} - \frac{n_2\pi}{2}\right)\right]\\ &\left[\beta_z G_6\cos\left(\beta_z z - \frac{\pi}{4} - \frac{n_3\pi}{2}\right) - \beta_z G_5\sin\left(\beta_z z - \frac{\pi}{4} - \frac{n_3\pi}{2}\right)\right]\hat{y}\\ &-\left[G_1\cos\left(\beta_x x - \frac{\pi}{4} - \frac{n_1\pi}{2}\right) + G_2\sin\left(\beta_x x - \frac{\pi}{4} - \frac{n_1\pi}{2}\right)\right]\\ &\left[\beta_y G_4\cos\left(\beta_y y - \frac{\pi}{4} - \frac{n_2\pi}{2}\right) - \beta_y G_3\sin\left(\beta_y y - \frac{\pi}{4} - \frac{n_2\pi}{2}\right)\right]\\ &\left[G_5\cos\left(\beta_z z - \frac{\pi}{4} - \frac{n_3\pi}{2}\right) + G_6\sin\left(\beta_z z - \frac{\pi}{4} - \frac{n_3\pi}{2}\right)\right]\hat{z}\Bigg\}\end{aligned} \tag{21}$$

In this case boundary conditions are $\mathbf{E}_y = 0$ at $z = z_0$, $z = z_0 + c$, $x = x_0$ and $x = x_0 + {}_a$. Using the condition $Ey = 0$ at $x = x_0$, $x = x_0 + a$, $G_1 = 0$ and $G_2 \neq 0$,

$$x_0 = \frac{N\pi + \frac{\pi}{4} + \frac{n_1\pi}{2}}{\beta_x} \quad (N = 1,2,3\ldots) \qquad \beta_x = \frac{L\pi}{a} \quad (L = 1,2,3\ldots) \tag{22}$$

Similarly, from the condition $\mathbf{E}_y = 0$ at $z = z_0$, $z = z_0 + c$, $G_6 = 0$ and $G_5 \neq 0$,

$$\beta_z = \frac{P\pi}{c} \quad (P = 1,2,3\ldots) \tag{23}$$

for applying condition on z-component $\mathbf{E}_z = 0$ at $y = y_0$, $y = y_0 + b$, $G_4 = 0$ and $G_3 \neq 0$,

$$\beta_y = \frac{Q\pi}{b} \quad (Q = 1,2,3\ldots) \tag{24}$$

The wave number for TE mode,

$$\beta = \sqrt{\left(\frac{L\pi}{a}\right)^2 + \left(\frac{Q\pi}{b}\right)^2 + \left(\frac{P\pi}{c}\right)^2} \tag{25}$$

and finally, the magnetic and electric field inside Menger sponge are calculated as,

$$H_x(x,y,z) = Kx^{n_1-\frac{1}{2}}y^{n_2-\frac{1}{2}}z^{n_3-\frac{1}{2}}\sin\left(\frac{L\pi}{a}x-\frac{\pi}{4}-\frac{n_1\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y-\frac{\pi}{4}-\frac{n_2\pi}{2}\right)\cos\left(\frac{P\pi}{c}z-\frac{\pi}{4}-\frac{n_3\pi}{2}\right) \tag{26}$$

$$E(x,y,z) = -\frac{j}{\omega\epsilon}Kx^{n_1-\frac{1}{2}}y^{n_2-\frac{1}{2}}z^{n_3-\frac{1}{2}} \times \left[-\beta_z\sin\left(\frac{L\pi}{a}x-\frac{\pi}{4}-\frac{n_1\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y-\frac{\pi}{4}-\frac{n_2\pi}{2}\right)\sin\left(\frac{P\pi}{c}z-\frac{\pi}{4}-\frac{n_3\pi}{2}\right)\right]\hat{y} + \left[\beta_y\sin\left(\frac{L\pi}{a}x-\frac{\pi}{4}-\frac{n_1\pi}{2}\right)\sin\left(\frac{Q\pi}{b}y-\frac{\pi}{4}-\frac{n_2\pi}{2}\right)\cos\left(\frac{P\pi}{c}z-\frac{\pi}{4}-\frac{n_3\pi}{2}\right)\right]\hat{z} \tag{27}$$

where $K = G_2G_3G_5$ and (29) , (30) are the electric and magnetic field equations, respectively for PEC boundary. For $n_1 = n_2 = n_3 = 0.5455$, (29) becomes,

$$H_x(x,y,z) = Kx^{0.0455}y^{0.0455}z^{0.0455}\sin\left(\frac{L\pi}{a}x-\frac{\pi}{4}-\frac{0.5455\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y-\frac{\pi}{4}-\frac{0.545\pi}{2}\right)\cos\left(\frac{P\pi}{c}z-\frac{\pi}{4}-\frac{0.545\pi}{2}\right) \tag{28}$$

classical results can be recovered [18], when the dimension is integer i.e., $D = 3$ and $\alpha_1 = \alpha_2 = \alpha_3 = 1$. The equation of magnetic field becomes,

$$H(x,y,z) = H_x(x,y,z) = K\sin\left(\frac{L\pi}{a}x-\frac{\pi}{4}-\frac{n_1\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y-\frac{\pi}{4}-\frac{n_2\pi}{2}\right)\cos\left(\frac{P\pi}{c}z-\frac{\pi}{4}-\frac{n_3\pi}{2}\right) \tag{29}$$

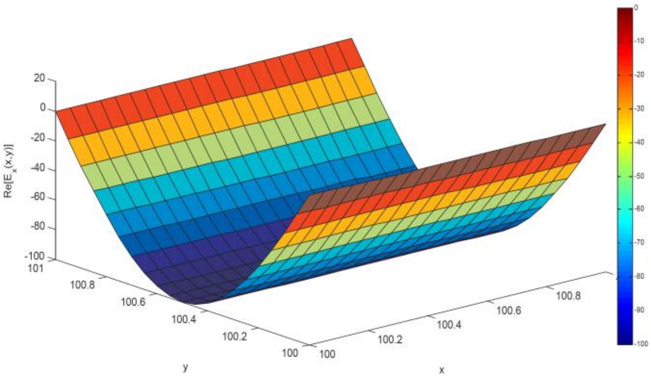


Fig. 2. Electric field in integer dimension spaces,TM$^X_{01}$.

## IV. Simulation Results

Eq. (16) is electric field equation for non-integer dimensional space. If variation along $z$ direction is kept constant it gives,

$$E_x(x,y,z) = C'x^{n_1-\frac{1}{2}}y^{n_2-\frac{1}{2}}\sin\left(\frac{L\pi}{a}x-\frac{\pi}{4}-\frac{n_1\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y-\frac{\pi}{4}-\frac{n_2\pi}{2}\right) \tag{30}$$

for integer-dimensional space i.e., $\alpha_1 = \alpha_2 = 1$, this problem reduces to classical results which are same as those mentioned in Balanis given as [18],

$$E_x(x,y,z) = C''\sin\left(\frac{L\pi}{a}x-\frac{\pi}{2}\right)\cos\left(\frac{Q\pi}{b}y-\frac{\pi}{2}\right) \tag{31}$$

where $C'$ and $C''$ are constants. Plots of electric field with $D$ = 3 and $D$ = 2.727 were obtained for comparison in integer and fractional space. Fig. 2 and Fig. 3 show the electric field for TM$^x_{01}$ mode and TM$^x_{11}$ mode, respectively against varying distance for integer dimension ($D$ = 3). Similarly, same modes, TM$^x_{01}$ mode and TM$^x_{11}$, are plotted for Menger sponge (fractal medium with $D$ = 2.727) in Fig. 4 and Fig. 5, respectively as a function of distance.

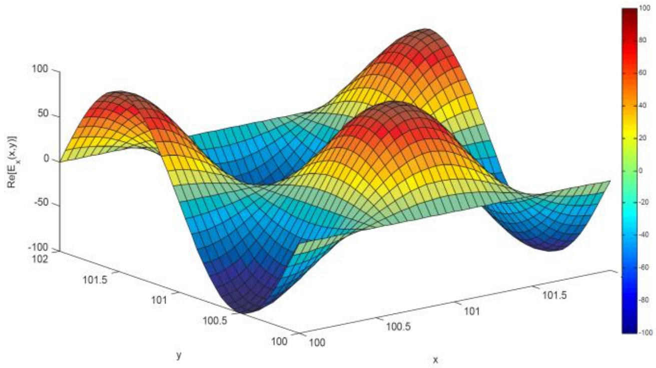


Fig. 3. Electric field in integer dimension spaces,TM$^X_{11}$.

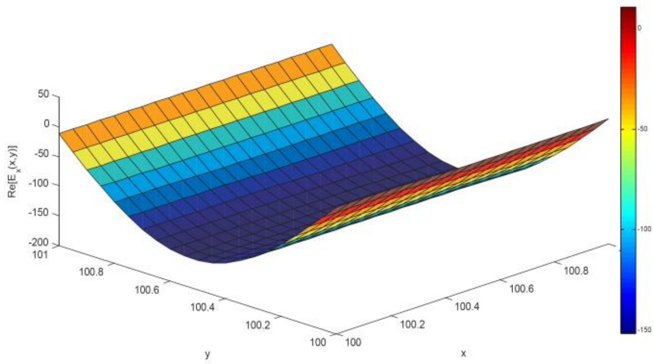


Fig. 4. Electric field in fractional space,TM$^X_{01}$.

## V. Conclusion

TM and TE modes of Menger sponge have been derived in this work using wave equations of fractional space. The modes were calculated under two conditions of boundaries i.e., PMC and PEC. The results show that the wave number is same for both cases and the duality principle holds for each case. It is also found that the classical results could be recovered when the integer dimension is considered. Moreover, these solutions can also be used for other fractal cubes.

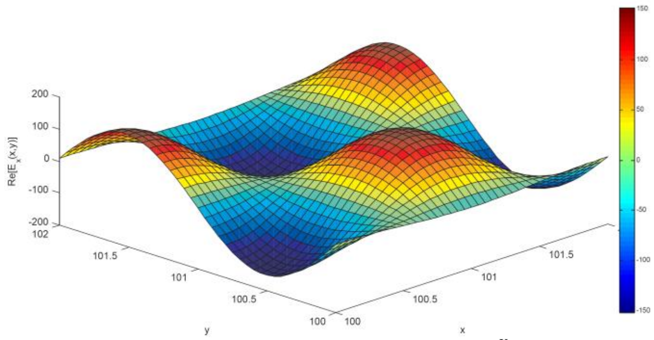


Fig. 5. Electric field in fractional space, $TM^{X}_{11}$.

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